Development of Soft Set Theory

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Abstract

In this paper, we give a crisp and critical survey of the development of soft set theory and enumerate some of its various applications in different direction to date. In particular, the work demonstrates that soft set theory can be applied to problems that contain uncertainties especially in decision making problems. These applications explain the voluminous work in this field within a short period of time. We emphasize that soft set has enough developed basic supporting structures through which various algebraic structures in theoretical point of view could be developed.

Keywords: Soft set, Fuzzy soft set, parameterization, soft BCK–algebras and soft group

1. *Introduction*

Most of our real life problems in engineering, social and medical science, economics, environment etc. involve imprecise data and their solutions involve the use of mathematical principles based on uncertainty and imprecision. To handle such uncertainties, a number of theories have been proposed. Some of these are probability, fuzzy sets, intuitionistic fuzzy sets [15], interval mathematics and rough sets etc. All these theories, however, are associated with an inherent limitation, which is the inadequacy of the parameterization tool associated with these theories.Some inherent problems of probability theory and interval mathematics can be found in [5].

Fuzzy set was developed by L. A.Zadehin (1965), in an attempt to deal with the problems of uncertainties. This theory has been found to be appropriate to some extent.

Let A be a subset of set X. $\mu_A$called indicator function, is defined as

$$\mu_A = \begin{cases} 1, if\ x \in A, \\ 0, if\ x \notin A. \end{cases}$$

Obviously there is one-to-one correspondence between a set and its indicator function.
Let U be a universe. A fuzzy set X, over U is a set defined by a function μ_X representing a mapping
$\mu_X: U \longrightarrow [0,1]$.

Here, $\mu_X$is called the membership function of X, and the value $\mu_X(u)$ is called the grade of membership of $u \in U$,andrepresents the degree of u belonging to the fuzzy set X. Thus, a fuzzy set X over U can be represented as follows:

$$X = \{(\mu_X(u)/u): u \in U, \mu_X(u) \in [0,1]\}$$

One of the deficiencies of fuzzy set theory is how to set the membership function [5].

Given the various observations on the efficiency of the existing tools for solving uncertainty problems, soft set theory emerged to soften these limitations.

2. *Concept of Soft Set*

Soft set is a parametrized general mathematical tool which deal with a collection of approximate descriptions of objects. Each approximate description has two parts, a predicate and an approximate value set. In classical mathematics, a mathematical model of an object is constructed and define the notion of exact solution of this model. Usually the mathematical model is too complicated and the exact solution is not easily obtained. So, the notion of approximate solution is introduced and the solution is calculated. In the soft set theory, we have the opposite approach to this problem. The initial description of the object has an approximate nature, and we do not need to introduce the notion of exact solution. The absence of any restrictions on the approximate description in soft set theory makes this theory very convenient and easily applicable in practice. Any parameterization we prefer can be used with the help of words and sentences, real numbers, functions, mappings and so on.

Definition: Soft set [5]

Let U be a universal set and let E be a set of parameters (each parameter could be a word or a sentence). Let $P(U)$denotes the power set of$U$.
A pair (F, E) is called a *soft set* over a given universal set U, if and only if F is a mapping of a set of parameters E, into the power set of U. That is, $F : E \longrightarrow P(U)$. Clearly, a soft set over U is a parameterized family of subsets of a given universe U. Also, for any $e \in E$, $F(e)$ is considered as the set of e–approximate element of the soft set(F, E).

Soft sets could be regarded as neighbourhood systems, and they are a special case of context-dependent fuzzy sets. In soft set theory the problem of setting the membership function, among other related problems, simply does not arise. This makes the theory very convenient and easy to apply in practice as we emphasized in section 3 of this paper.

Example 1

(i) Let (X, τ) be a topological space, that is, X is a set and τ is a toplogy (a family of subsets of X called the open sets of X). Then, the family of open neighbourhoods $T(x)$ of point x, where $T(x) = \{V \in \tau \,|x \in V\}$, may be considered as the soft set $(T(x), \tau)$.

(ii) Let A be a fuzzy set and μ_A be the membership function of the fuzzy set A, that is, μ_A is a mapping of U into $[0, 1]$, let $F(\alpha) = \{x \in U \,|\mu_A(x) \geq \alpha\}, \alpha \in [0, 1]$ be a family of $\alpha -$ level sets for function μ_A. If the family F is known, $\mu_A(x)$ can be found by means of the definition: $\mu_A(x) = Sup_{\substack{\alpha\in[0,1],\\ x\in F(\alpha)}}\ \alpha$. Hence every fuzzy set A may be considered as the soft set $(F, [0, 1])$.

(iii Let $U = \{C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9, C_{10}\}$ be the set of Cars under consideration, E be a set of parameters.
$E = \{e_1 =$ expensive, $e_2 =$ beautiful, $e_3 =$ manual gear, $e_4 =$ cheap, $e_5 =$ automatic gear, $e_6 =$ in good repair, $e_7 =$ in bad repair$\}$.Then the soft set (F, E) describes the attractiveness of the cars.

3. *Various Applications of Soft Set*

Soft set theory has potential applications in many different fields which include the smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability theory, and measurement theory, see [5] for details.

In [5] applications of soft set to stability and regularization, game theory and operational research are discussed in details. Extension of soft set theory to real analysis and its applications were also presented. Also, application of soft set theory to the problems of medical diagnosis in medical expert system was discussed in [5].

Applications of soft set theory in other disciplines and real life problems are now catching momentum. Maji*et al.*[24], in year 2002, gave first practical application of soft sets in decision making problems. It is based on the notion of knowledge of reduction in rough set theory.

Soft set theoretic approach for dimensionality reduction and fuzzy parameterized fuzzy soft set theory and its applications were discussed in [23].Soft set theory is applied to commutative ideals in BCK-algebras. The notion of commutative soft ideals and commutative idealistic soft BCK-algebras are introduced in [30]. Example to show that there is no relations between positive implicative idealistic soft BCK-algebras and commutative idealistic soft BCK-algebras is provided.

In [7] the concept of soft relations and fuzzy soft relations were discussed and applied in a decision making problem.Soft set theory based classification algorithm was proposed in [19] which can be applied to texture classification. This proposed algorithm has very low computational complexity when compared with Bayes classification technique and yield very good classification accuracy,see [19] for details.In [8] soft set theory is used to initiate the study of semirings. The notions of soft semirings, soft subsemirings, soft ideals, idealistic soft semirings and soft semiringhomomorphisms are introduced and several related properties are investigated. On the other hand, semirings have been found useful for dealing with problems in different areas of applied mathematics and information sciences, as the semiring structure provides an algebraic framework for modeling and investigating the key factors in these problems. The application of semirings to areas such as optimization theory, graph theory, theory of discrete event dynamical systems, generalized fuzzy computation, automata theory, formal language theory, coding theory and analysis of computer programs have been extensively studied in the literature.

In [27] an alternative approach for attribute reduction in multi-valued information system under soft set theory is presented. Based on the notion of multi-soft sets and AND operation, attribute reduction is defined. Application of soft set theory have been extended to the concept of bijective soft set and some of its operations such as the restricted AND and the relaxed AND operations on a bijective soft set, dependency between two bijective soft sets,bijective soft decision system, significance of bijective soft set with respect tobijective soft decision system and reduction of bijective soft decision system were extensively discussed, [17]. With these notions and operations, an application of bijective soft decision soft set in decision-making problems is revealed.

Soft set approaches have been extended to the application of the concept of intuitionistic fuzzy soft sets oversemigroup theory. In [24], the notion of intuitionistic fuzzy soft ideals over a semigroup is introduced and their basic properties are investigated. Some lattice structures of the set of all intuitionistic fuzzy soft ideals of a semigroupwerealso derived.

4. Historical Perspective

The origin of soft set theory could be traced to the work of Pawlak [32] in 1993 titled *Hard and Soft Set* in Proceeding of the International EWorkshop on rough sets and knowledge discovery at Banff. His notion of soft sets is a unified view of classical, rough and fuzzy sets. This motivated D. Molodtsov's work [5] in 1999 titled *soft set theory: first result*.Therein, the basic notions of the theory of soft sets and some of its possible applications were presented. For positive motivation, the work discusses some problems of the future with regards to the theory.

In order to solidify the theory of soft set, P. K. Maji*et al.*, [24] in 2002, defined some basic terms of the theory such as equality of two soft sets, subset and super set of a soft set, complement of a soft set, null soft set, and absolute soft set with examples. Binary operations like AND, OR, union and intersection were also defined. De Morgan's laws and a number of results are verified in soft set theory context. A. Sezgin and A. O. Atagun [3] proved that certain De Morgan's law holds in soft set theory with respect to different operations on soft sets.Ali*et al.*, [19] in 2009, introduced some new notions such as the restricted intersection, the restricted union, the restricted difference and the extended intersection of two soft sets. They improved the notion of complementation in soft set and also proved that certain De Morgan's' law hold in soft set theory.

In [24] also, soft set theory was applied to solve a decision making problem using rough set theory and an algorithm to select the optimal choice ofan object was provided. This algorithm uses fewer parameters to select the optimal object for a decision problem. However, in decision making problem in [24], there is a straightforward relationship between the decision values of objects and the conditional parameters. That is, the decision values is computed with respect to the conditional parameters. This is quite different in the case of rough sets.In rough set theory the decision attributes are not computed according to the conditional attributes.

K. V. Babitha and J.J Sunil [16] in 2010, introduced the concept of soft set relations as a sub soft set of the cartesian product of the soft sets and many related concepts such as equivalent soft set relation, partition, composition and function are discussed.Also,A.Kharal and B. Ahmad [4] defined mapping on soft classes and studied several properties of images and inverse images of soft sets supported by examples and counterexamples. These notions were applied to the problem of medical diagnosis in medical expert systems.

N. Cagman and S. Enginoglu [23] in 2010, defined soft matrices and their operations to construct a soft max-min decision making method which can be successfully applied to the problems that contain uncertainties.

D. Chen *et al.*[19] in 2005, focus their discussion on the parameterization reduction of soft sets and its applications. First they pointed out that the results of soft set reductions offered in [24] are not correct. They also observe that the algorithms used to first compute the choice value to select the optimal objects for the decision problems are not reasonable and they illustrate this with an example. Finally, they proposed parameterization reduction of soft sets and compared it with the concept of attributes reduction in rough sets theory.

H.Aktas and N.Cagman[11] in 2007 extended the basic properties of soft sets in [24] and comparedthe soft sets to the related concept of fuzzy sets and rough sets. They then gave a definition of soft groups, and derived their basic properties.In 2008, A. Karadogan [1] work on the application of fuzzy soft set theory in the selection of underground mining method. B. Ahmad and A.Kharal [4] in 2009, further contribute to the properties of fuzzy soft setsand support them with examples. They also definedsome operations on fuzzy soft set like union and intersection and proved some basic properties.

F.Feng*et al.*, [8] in 2008extended the study of soft set to soft semirings. The notions of soft semirings, soft subsemirings, soft ideals, idealistic soft semirings and soft semiringhomomorphismswere introduced, and several related properties were investigated.The theory of BCK/BCI–algebras is now being studied via soft set theory [31].

T. Herawan and M. M. Deris [27] in2009,did a comparative study between rough sets and soft sets and presented a direct proof that Pawlak's and Iwiki's rough sets can be considered as soft sets.

M. M. Mushrif [21] in 2006 studied the texture classification via soft set theory.In 2010, Wei Xu*et al.*[28] introducd the notion of vague soft set which is an extension of soft set. The basic properties of vague soft sets are presented and discussed.

K. Qin and Z. Hong[18] dealt with algebraic structure of soft sets. The lattice structures of soft sets were constructed. The concept of soft equality was introduced and some related properties were derived. It is proved that soft equality is a congruence relation with respect to some operations and the soft quotient algebra wasintroduced.Y. B. Jun *et al.*, [31] in 2008 applied the notion of soft sets to commutative ideals of BCK–algebras. Implicative soft ideals and commutative soft ideals were also investigated.

T. Herawan*et al.*,[27] in 2010, gave an alternative approach for attribute reduction in multi-valued information system under soft set theory. The work emphasized that based on the notion of multi-soft sets and AND operation, attribute reduction can be defined. It is shown that the reducts obtained are equivalent with Pawlak's rough reduction.

N.Cagman*et al.*,[22] in 2010,extended soft set to fuzzy parameterized fuzzy soft sets and explicate their applications indecision making method. Also in the same year, K. Gong*et al.*, [17], proposed the concept of bijective soft set and some of its operations were discussed.Finally, an application of bijective soft set in decision-making problems was also discussed.

In 2011, N.Cagman and S.Enginoglu [22] redefinedsome of the operations of soft sets to make them more functional to improve several new results. By using these new definitions they also construct a decision making method which selects a set of optimum elements from the alternatives. Also, in the same year, X.Ge and S. Yang[29] further investigate the operational rules given by P. K.Maji*et al.*,[24] and M. I. Ali *et al.*,[19]. They obtain some necessary and sufficient conditions that made corresponding operational rules to hold. Y. B. Jun*et al.*, [30] applied the notion of soft sets to the theory of BCK-algebras.

The notions of positive implicative soft ideals and positive implicative idealistic soft BCK-algebra were introduced and their basic properties were derived.S.Alkhazaleh*et al*.,[26]extended soft set to soft multisetand present its basic operations such as complement, union and intersection.

M. A. Ozturk and E. Inan [20] in 2011illustrate the interconnections between the various operations in soft set and defined the notion of restricted symmetric difference of soft sets and investigated its properties.Also, S. V. Manemaran[25] in 2011 further discussed fuzzy soft sets algebraic structures and defined fuzzy soft group. Operations on fuzzy soft groups and some related results were proved. Furthermore, definitions of fuzzy soft functions and fuzzy soft homomorphism are also defined. Finally, the theorems on homomorphic image and homomorphic pre image werediscussed in detail. J. Ghosh*et al*., [13]extended soft set to ring theory and also present fuzzy soft ideal theory. H. Prade and D. Dubois [9] work on the theory of fuzzy sets and system with relevant application.

A. O.Atagun and A.Sezgin [2] in 2011studied soft subrings and soft ideals of a ring. Moreover, the concept of soft subfields of a field and soft submodule of a left R-module were defined. Some related properties about soft substructures of rings, fields and modules are investigated and illustrated by some examples. Soft set is being currently extended to intuitionistic fuzzy soft sets and the concept of intuitionistic fuzzy soft sets to semi group theory. The notion of intuitionistic fuzzy soft ideals over a semi group is introduced with their basic properties investigated, J. Zhou *et al*., [14]. Also, some lattice structures of the set of all intuitionistic fuzzy soft ideals of a semi group were derived.

N. Cagman*et al*. [23] also extendedsoft set to fuzzy parameterized (fp) soft sets and proposed a decision making method based on FP-soft set theory. With examples, they showed that the method can be successfully applied to the problems that contain uncertainties.

H. Yang and Z.Guo[12] studied the concept of anti-reflexive kernel, symmetric kernel, reflexive closure, and symmetric closure of a soft set relation. Finally, soft set relation mapping and inverse soft set relation mappings were proposed and some related properties were discussed.D. K.Sut[7] in 2012worked on soft set relations and extends them to fuzzy soft relations and also applied it in a decision making problem.

5. Conclusions

We crisply summarized the basic concepts of soft set theory and enumerate some of its various applications in different direction to date. Soft set isan instrument for dealing with uncertainty problems. Its efficiency in dealing with uncertainty problems is as a result of its parameterized concept. These applications explain the voluminous work in this field within a short period of time. Finally, soft set has enough developed basic supporting structures. Many algebraic structures could be developed via this field.

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