Evolutionary Topological Design of Two Dimensional Composite Structures

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Abstract

This paper presents topological design of two dimensional composite structures using evolutionary fully stressed design technique. The basic idea of this method is to reach to a topology where the stress distribution is uniform under the external loading. In order to achieve this, material is removed from the low stressed parts while new material is added to the higher stressed regions. The iterative topological process is initiated by first discretizing the two dimensional design space by eight node solid elements. Finite element analysis of the system is then carried out under the external loading and maximum principle stresses are calculated. The principal stresses are compared with the threshold stress value and elements that have lower stress values are removed from the design space. New elements are added to the areas where the stresses have higher values. If stresses exceed the new material limit stresses, the new reinforcement elements are added to area which is carried out highest stresses value. During this process the optimum topology of the structure slowly emerges.

Keywords: Evolutionary design, fully stressed design, topology optimization, composite structure, finite element structures

1. Introduction

Topological design of structures constitutes the important part of structural optimization. It is possible to achieve more economical designs by topological optimization than by only carrying out size optimization. The number of algorithms developed which treats the topology of the structure as design variable on the other hand, is much more less than the number of structural optimization techniques that determines optimum cross-sectional properties only. One reason behind this is that the topological design problem is much more complex and highly nonlinear than that of size optimization problem. Obtaining the solution of such problem by mathematical programming or optimality criteria approaches is cumbersome and presents numerical difficulties. It requires expertise that is beyond the ability of everyday practicing structural engineer.

In recent years new branch of optimization techniques were introduced which mimics the design methods exists in nature. Genetic algorithms, simulated annealing and evolutionary strategies are among such algorithms that are used in the topological design of structures (M. Osaki,2002 and Y.M. Xie, G.P. Steven, C. Mattheck, 1997). Among these genetic algorithms are search methods that are based on the principal of the survival of the fittest and adaptation. They operate on a population of design variables set. Each population consists of individuals that are potential solution to the design problem. A fitness value is calculated for each individual using the objective function and constraints as a measure of performance of the design variables. If the individual is fit, it is selected as candidate to take part in the construction of the next population, if not then it is let to die off. Simulating annealing is a stochastic technique that is based on the analogy to the physical process of annealing a metal. The solution to a general optimization process can be associated with this system behavior. The cost of a structure corresponds to the concept of energy and moving to any new set of design variables corresponds to a change of state.

Evolutionary design method on the other hand is the application of the principle of the adaptive growth to structural optimization. The adaptive growth process observed in the nature aims at producing a structure for the living beings such that it has a uniform stress distribution. It simply achieves this by depositing material at the overloaded zones and not depositing material as is the case in tress or even reduction of material as is the case in bones at the under-loaded zones. Repeating this process during, the growth of living being results in the most efficient use of the material in its construction. Furthermore, this process finally arrives at a structural shape that is fully stressed under the loads that living being is subjected to. This concept has been applied to determine the optimum topology or layout of structures by number of researchers using different titles.

The soft-kill method is one of the evolutionary design method suggested by Walther et al (F. Walther, C. Mattheck, 1993). In this method the modulus of elasticity of the element is simply set to the stress calculated at the particular element. The stress computed is either taken as maximum principle stress or the equivalent stress such as Von Mises stress. Once the modulus of elasticity of element is related to the stress develop in the element, it means a linear relationship is assumed between the modulus of elasticity and the stress in the element. This means that the highly loaded zones become harder, and the less loaded zones become softer. Thus the formerly homogeneous material becomes non-homogeneous. If stress calculation is carried out in this non-homogeneous structure, the strong load-bearing zones carry even more and previously unloaded zones carry even less. Repeating these stress computations iteratively result in those unloaded elements has stresses below a certain minimum value. When these stresses are set to zero and these elements are removed from the structure in the final stage the layout with uniform stress is obtained. Since the elements are removed gradually during the stress calculation this method is called soft-kill method.

Hinton and Sienz (E. Hinton, J Sienz, 1995) named the evolutionary design method they presented as hard-kill method, while Xie and Steven (Y. M. Xie, G. P. Steven , 1994) called the similar technique evolutionary structural optimization (ESO). In the hard-kill method, a step function is used instead of a linear function. The elements with stresses below a certain equivalent stress are assigned low modulus elasticity. In this way such elements virtually carry no load and their stress levels are small in subsequent analysis. The hard-kill method initially discretizes the design domain. It then carries out elastic finite element analysis with constant modulus of elasticity for all elements and determines the equivalent stress or maximum principle stress for each element. Those elements that have stress less then than a certain selected stress value are identified. The modulus of elasticity of these elements is equated to a relatively small value such as 10^{-6}E where E is the modulus of elasticity of the material. These elements are nor actually removed from the structure, but rather switched off so that they do not contribute in load carrying capacity of the structures. This way does not alter the finite element topology of the structure and eliminates the problem of instability. In this technique, it is also possible to allow the designer to switch on some of the elements that were previously switched off, if at certain stage stresses in these elements become larger than the specified. The iterative process of element removal and addition is continued until all stress levels become uniform.

In this study, topological design of two-dimensional composite structures is carried out using evolutionary design concept. For this purpose, two dimensional design space is discretized by eight node solid elements that has three degree of freedom at each node. Finite element analysis of the system is carried out under the external loading and maximum principle stresses and minimum principal stresses are calculated. The principal stresses are compared with the threshold stress values in tension and in compression and elements that have lower stress level are switched off. This process is repeated until the optimum topology gradually evolves.

2. Evolutionary Topological Design Algorithm

The evolutionary topological design of two-dimensional composite structures intends to obtain a structural topology that provides near-to-uniform stress distribution under the applied loads. This is achieved in an iterative process. The structure is analysed at each iteration, and the regions with low and high stresses are determined. Material is removed from regions with lower level of stresses and material is added to the regions with higher level of stresses. If stresses exceed the material limit stress, the new reinforcement elements are added to area which is carried out highest stresses value. This process is repeated until the volume of the structure does not significantly change within two iterations. The steps of the algorithm are given in the following.

1. The design space is discretized solid elements with eight-nodes.
2. Finite element analysis of the structure is carried out under the external loading and maximum and minimum principal stresses in each finite element are determined.
3. Element removal: Maximum and minimum principal stresses are compared with the global threshold stress value in each element and those that have both stress values lower than the global threshold value are determined. Such elements are removed from the structure. However not to cause a instability in the structure they are not actually removed from the design space but kept in the mesh with a reduced modulus of elasticity as suggested in (E. Hinton, J Sienz,1996). Hence the modulus of elasticity for a particular element to be used in iteration i+1 is determined as

$$E_{i+1} = E \quad \text{if} \quad \sigma_p > \sigma_t \tag{1a}$$
$$E_{i+1} = E_s \quad \text{if} \quad \sigma_p < \sigma_t \tag{1b}$$

4. where σ_p is the principal stress and σ_t is the global threshold value. E_{i+1} is the value of the modulus of elasticity for the particular element in iteration I+1, E is the modulus of elasticity of the material and E_s is the reduced modulus of elasticity. The value of E_s should be selected such that the contribution of the regions with this modulus of elasticity to load carrying capacity of the structure becomes negligible. Selection of too small value for this modulus of elasticity can cause ill-conditioning in the stiffness matrix. For the threshold stress small value is initially adopted and its value is increased linearly each iteration. It is found suitable in the design examples considered to take the initial value of threshold stress as 0.1N/mm^2 and it is increased each iteration by 0.1N/mm^2.
5. Element addition: The increase of threshold value during the iterations causes increase in the values of the maximum stresses. In the each iteration, the following ratio is computed for the maximum principal stress in the structure.

$$\frac{\left(\sigma_{p,i} - \sigma_{p,i-1}\right)}{\left(\sigma_{p,i-1} - \sigma_{p,i-2}\right)} \leq sirc$$

6. where i is the current iteration number, i-1 is the previous iteration number and i-2 is the iteration number, two iterations before the current. sirc is pre-selected value. σ_p is the maximum or minimum principal stress in the structure. It is found suitable to use value of 2 for sirc. If the above ratio for the maximum principal stress exceeds the value adopted for sirc for a particular element, then the neighboring elements to these elements are added to the mesh. In addition, if stresses exceed the new material limit stresses, the new reinforcement elements are added to area which is carried out highest stresses value. This reinforcement element of the modulus of elasticity has higher than once element of the modulus of elasticity. The above steps are repeated until no significant change is observed in the volume of the structure.

3. Design Examples

The evolutionary topological design algorithm presented above is applied to topological design of four examples. In all of these examples, the modulus of elasticity of the materials is taken as value 21kN/mm^2 and the modulus of elasticity of the reinforcement element is value 210kN/mm^2. The value of the modulus of elasticity E_s used for those elements that are to be switched off from the structure is $1\text{x}10^{-5}\text{KN/mm}^2$. The allowable stress of the first material is taken as ±140MPa in tension and compression and the second material is taken as ±240MPa in tension and ±140MPa compression. The threshold stress value is initially selected as 0.1MPa and is increased 0.1MPa in each design cycle.

All of the examples used michell type beam that have different load condition. The structure is considered in the design that has been discretized by 5000 solid elements with 3 degrees of freedom at each node. The Poisson's ratio and sıcr coefficient are taken as 0.15 and 1.5 respectively. The initial volume is $500\ \text{x}10^6\ \text{mm}^3$ and H, L, thickness are 5 m, 10 m, 0.1 m respectively, as shown in Fig. 1. (S. Savas, M. Ulker, M. P. Saka, 2003)

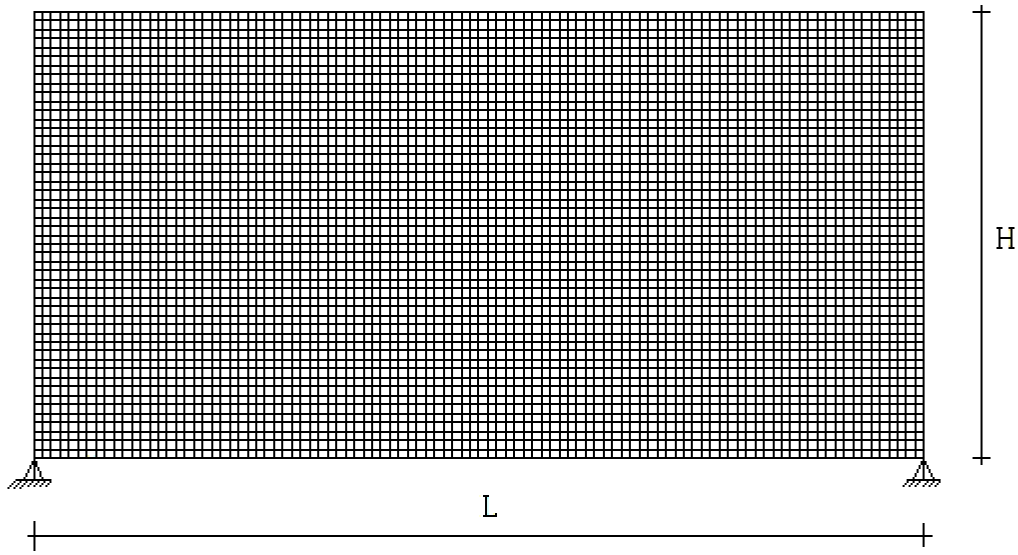


Fig.1 Design domain for a Michell type structure

3.1. Example of michell beam for 1 type load condition

This example of load condition is shown in Figs. 2. The other properties are identical. The topologies obtained during the iterations are illustrated in figures 3, 4, 5 and 6 clearly show the evolution of the optimum shape. The variation of the maximum and minimum principal stresses during the iterations is shown in Figs. 7 (a) and (b). These graphs clearly indicate the efficiency of the element removal and element addition algorithms used in the techniques. Both diagrams show the gradual increase of these stresses when the design technique moves from one topology to another until both principal stresses reach their upper bounds. Fig. 8 shows the variation of the volume of structures obtained during the topological design iterations. The minimum volume of the final topology is $114{,}5 \times 10^6$ mm^3. This value is less than volume of one material system that is $155{,}6 \times 10^6$ mm^3

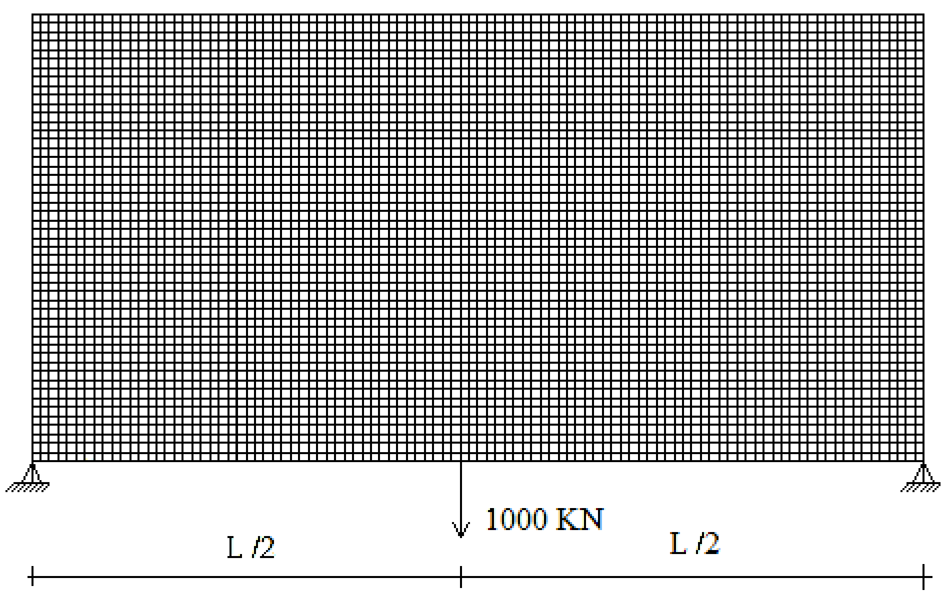


Fig. 2 1. type load condition

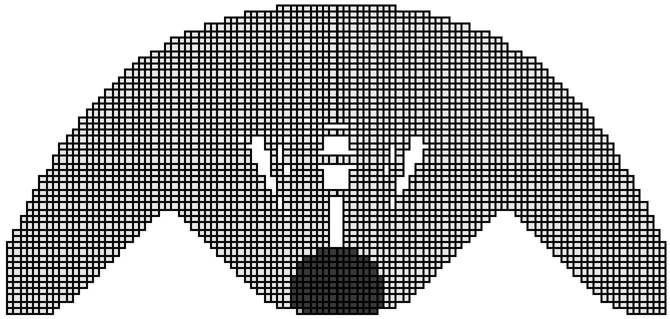


Fig 3. after 150 iteration

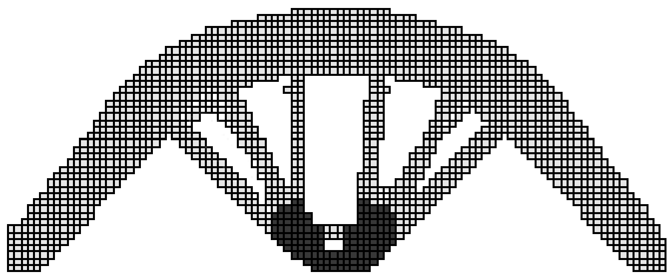


Fig 4. after 300 iteration

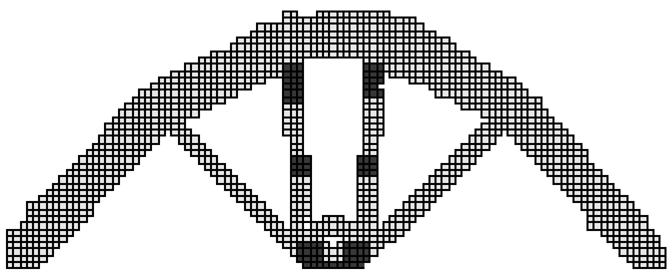


Fig 5. after 450 iteration

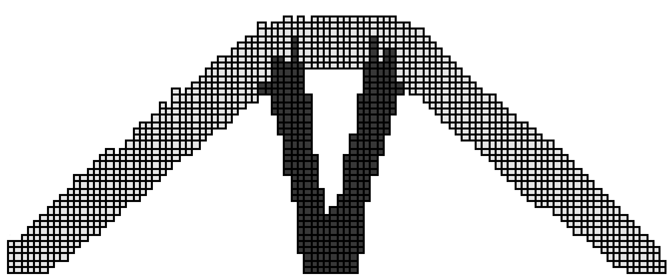


Fig 6. after 630 iteration

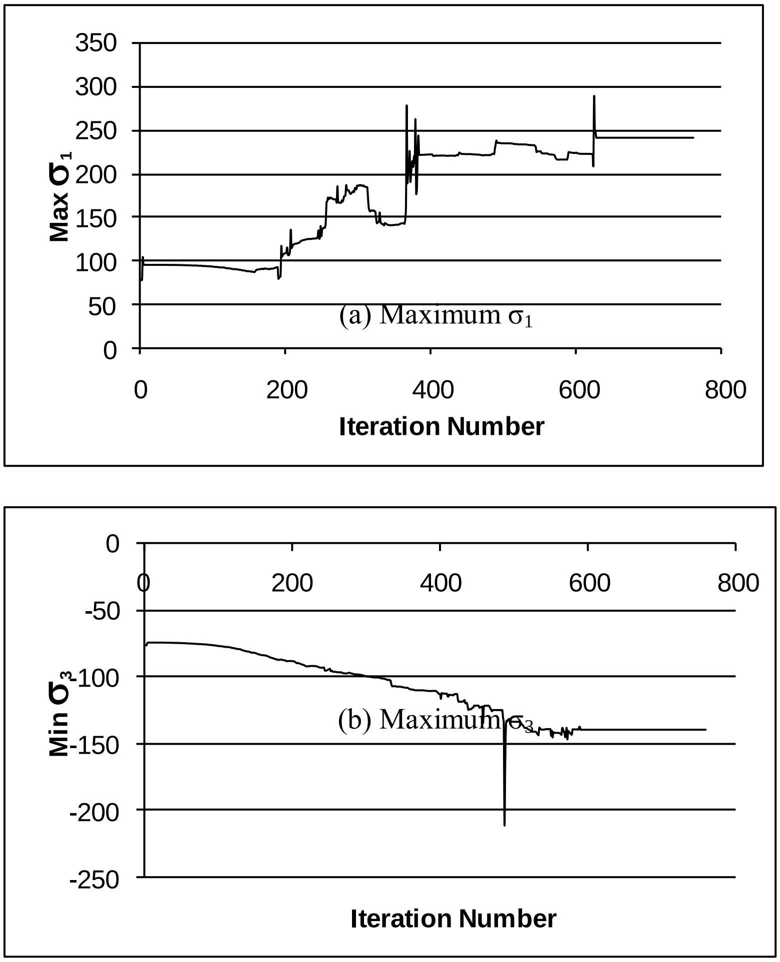


Fig. 7 Variation of the maximum principal stresses during the iterations

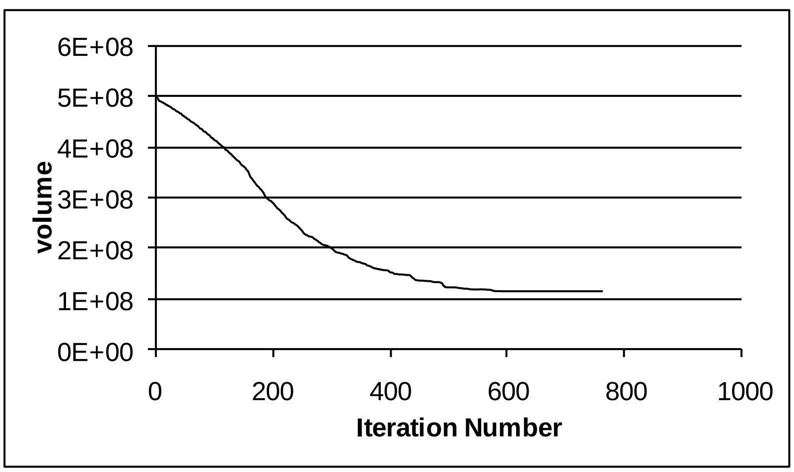


Fig. 8 Design history for the michell beam volume

3.2. Example of michell beam for 2 type load condition

This example of load condition is shown in Figs. 9. The other properties are identical. The topologies obtained during the iterations are illustrated in figures 10 and 11 clearly show the evolution of the optimum shape. The variation of the maximum and minimum principal stresses during the iterations is shown in Figs. 12 (a) and (b). Fig. 13 shows the variation of the volume of structures obtained during the topological design iterations. The minimum volume of the final topology is 258,2x10^6 mm^3.

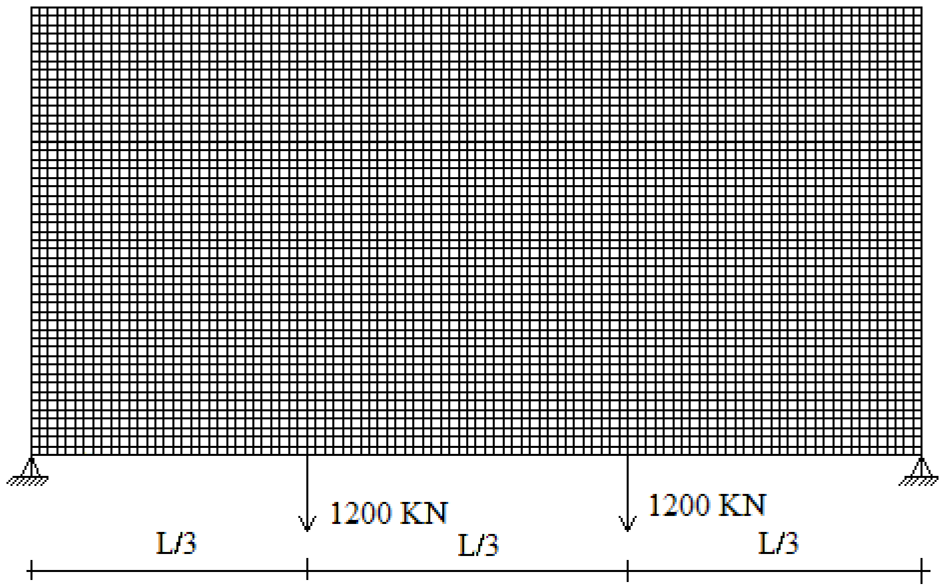


Fig. 9 2. type load condition

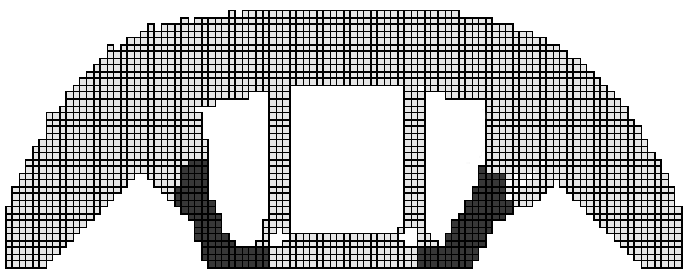


Fig 10. after 300 iteration

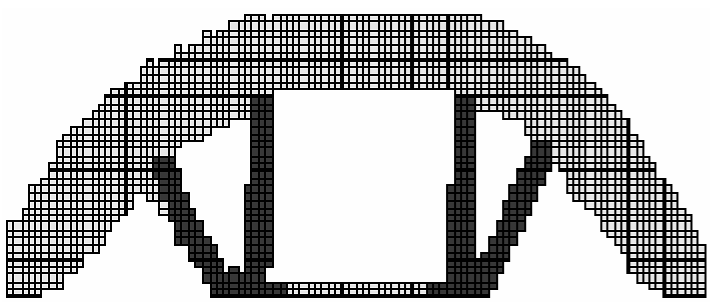
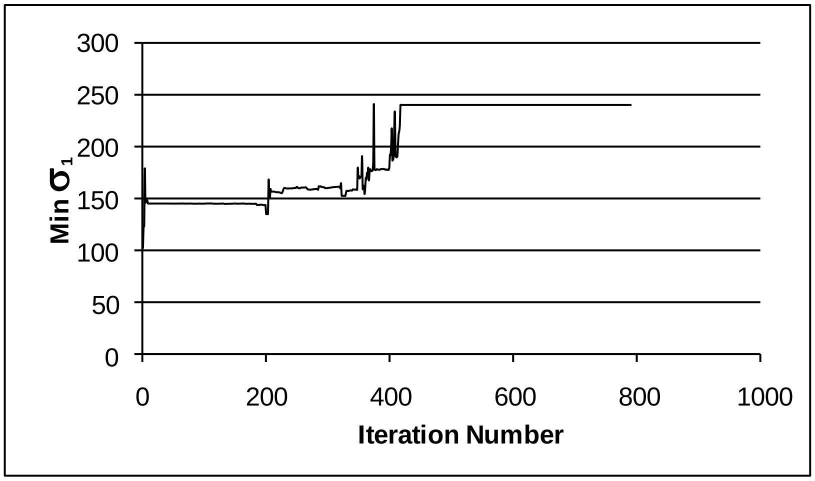
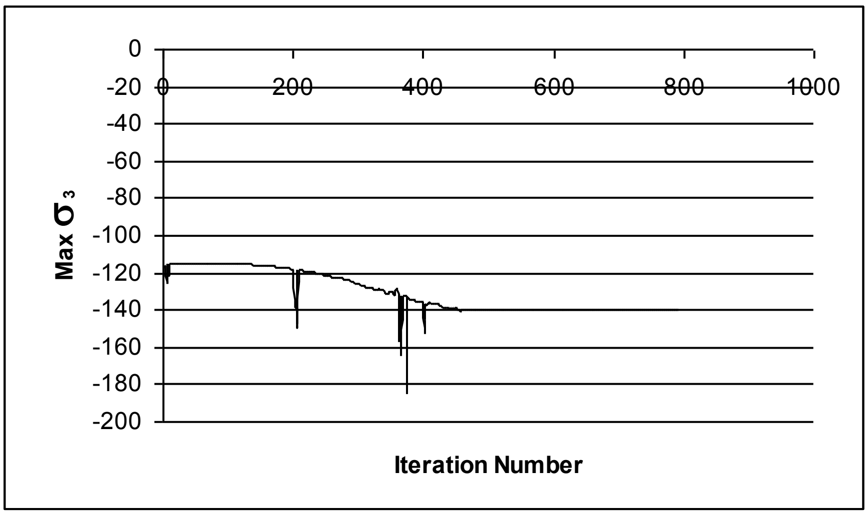


Fig 11. after 440 iteration



(a) Maximum σ_1



(b) Maximum σ_3

Fig. 12 Variation of the maximum principal stresses during the iterations

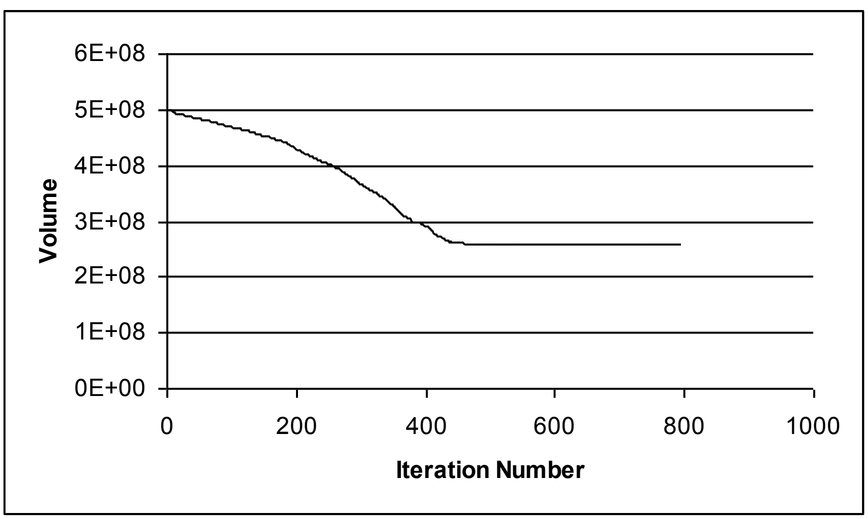


Fig. 13 Design history for the michell beam volume

3.3. Example of michell beam for 3 type load condition

This example of load condition is shown in Figs. 14. The other properties are identical. The topologies obtained during the iterations are illustrated in figures 15, 16, 17 and 18 clearly show the evolution of the optimum shape. The variation of the maximum and minimum principal stresses during the iterations is shown in Figs. 19 (a) and (b). Fig. 20 shows the variation of the volume of structures obtained during the topological design iterations. The minimum volume of the final topology is 151,6x10^6 mm^3.

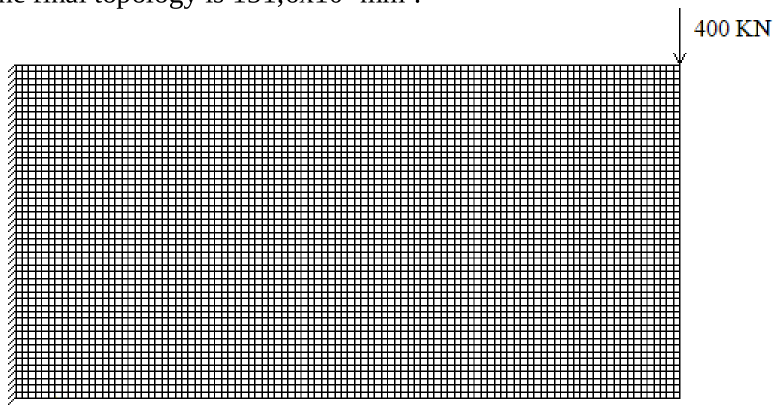


Fig. 14 2. type load condition

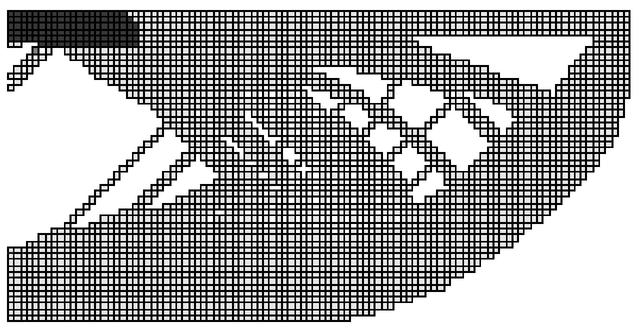


Fig 15. after 150 iteration

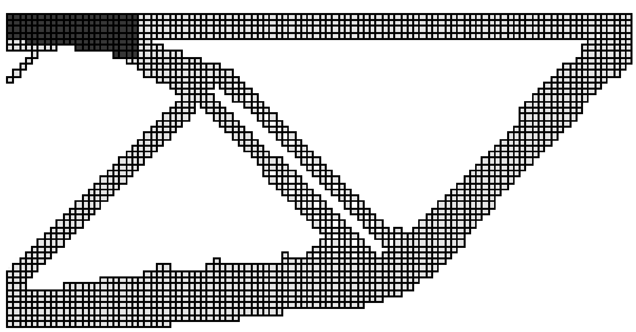


Fig 16. after 250 iteration

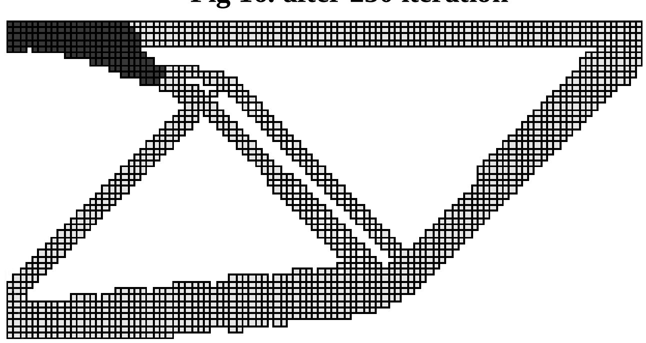


Fig 17. after 350 iteration

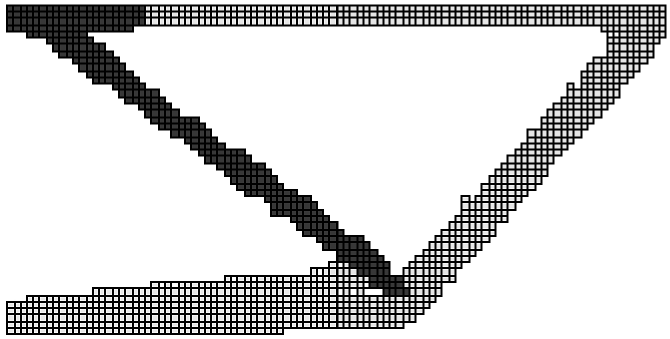
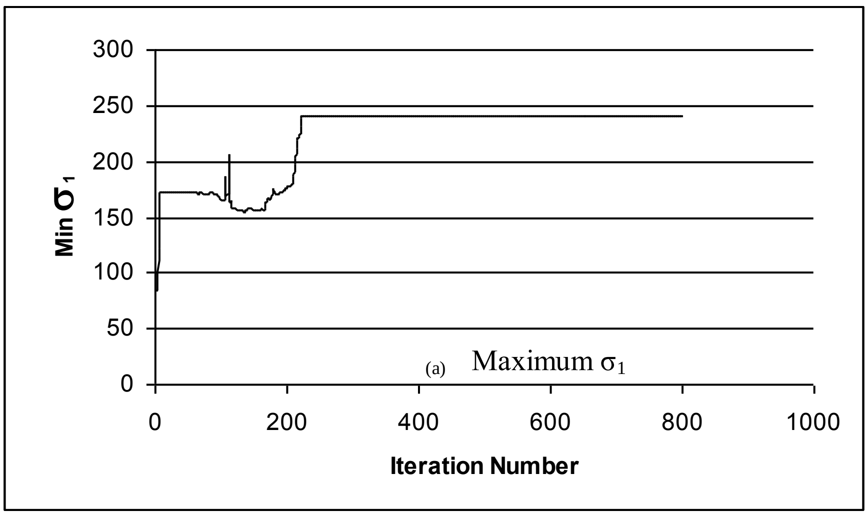
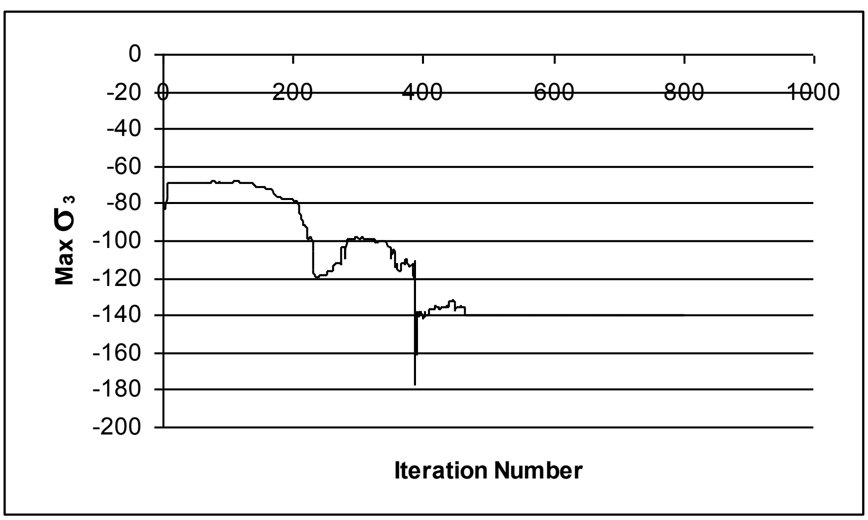


Fig 18. after 475 iteration



(a) Maximum σ_1



(b) Maximum σ_3

Fig. 19 Variation of the maximum principal stresses during the iterations

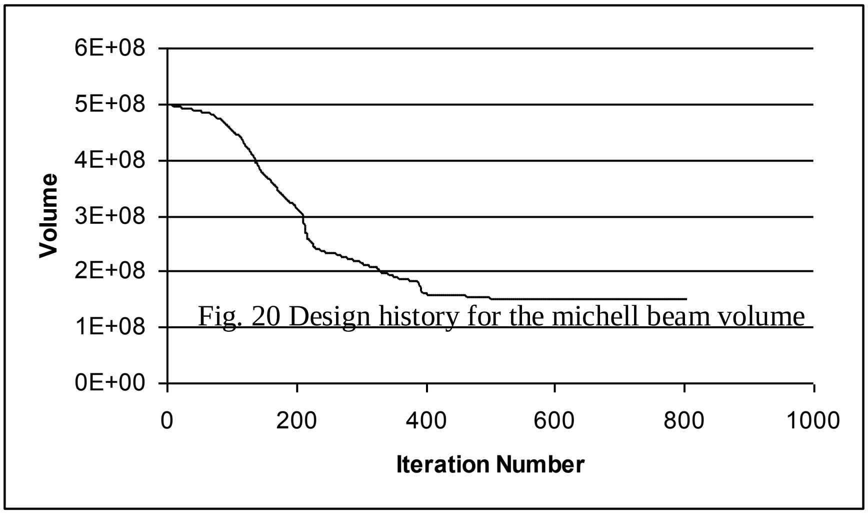


Fig. 20 Design history for the michell beam volume

3.4. Example of michell beam for 4 type load condition

This example of load condition is shown in Figs. 21. The other properties are identical. The topologies obtained during the iterations are illustrated in figures 22, 23 and 24 clearly show the evolution of the optimum shape. The variation of the maximum and minimum principal stresses during the iterations is shown in Figs. 25 (a) and (b). Fig. 26 shows the variation of the volume of structures obtained during the topological design iterations. The minimum volume of the final topology is 181,6x10^{6} mm^{3}.

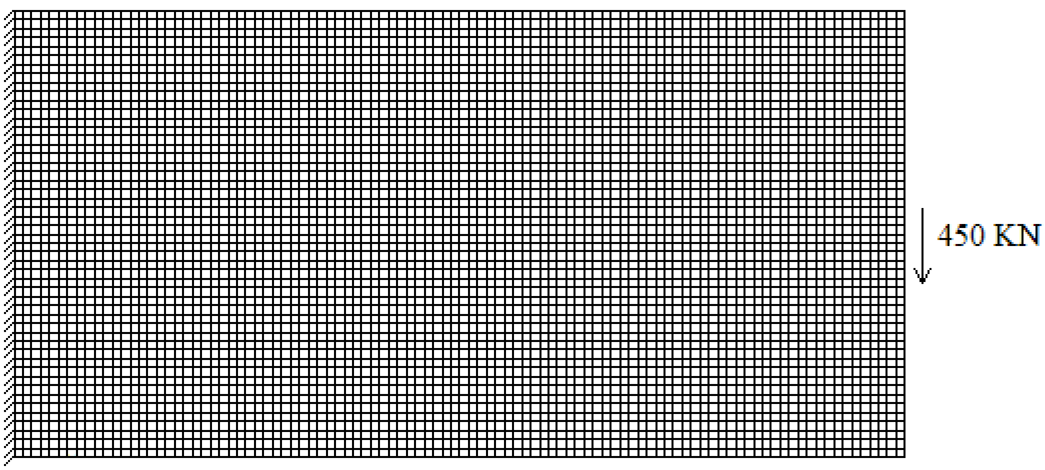


Fig. 21 2. type load condition

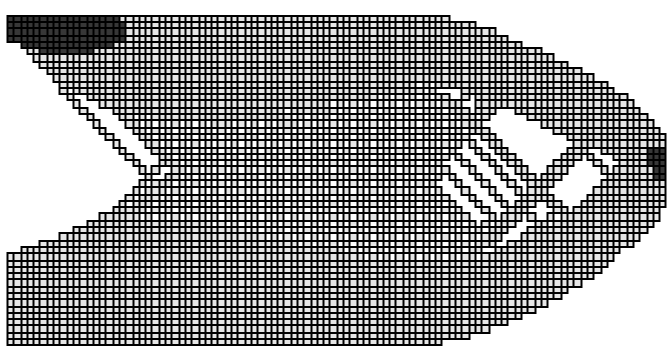


Fig 22. after 100 iteration

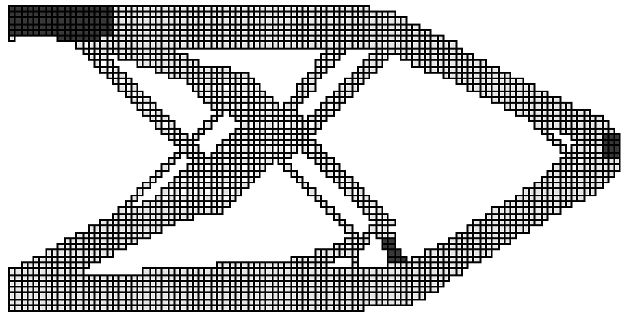


Fig 23. after 200 iteration

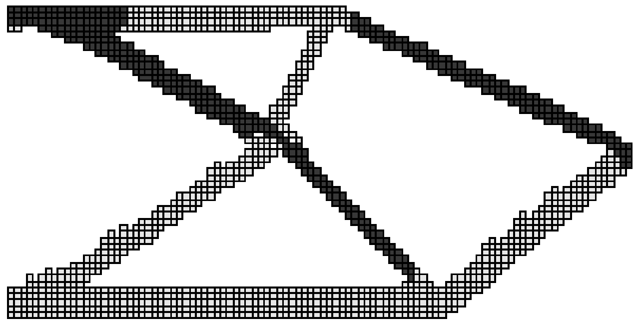
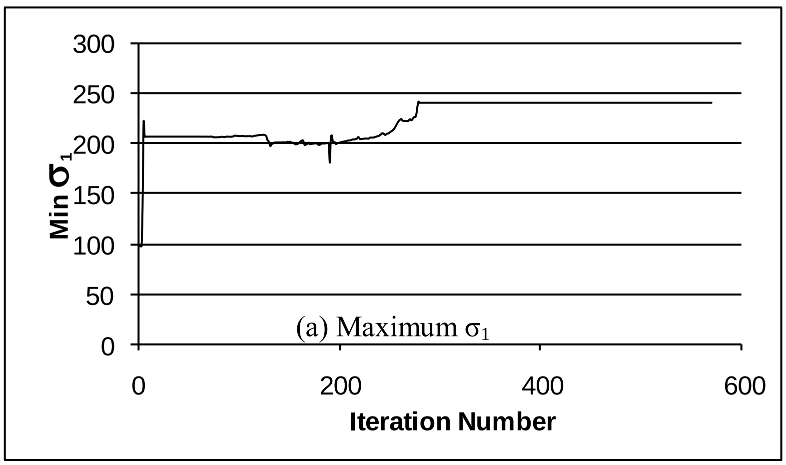


Fig 24. after 385 iteration



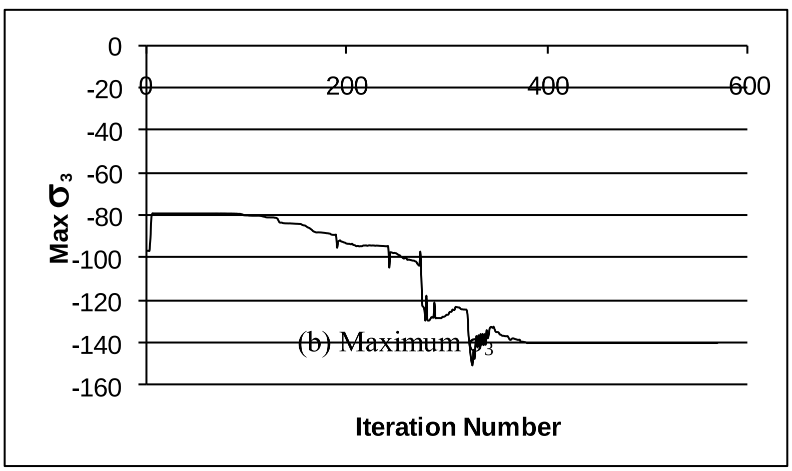


Fig. 25 Variation of the maximum principal stresses during the iterations

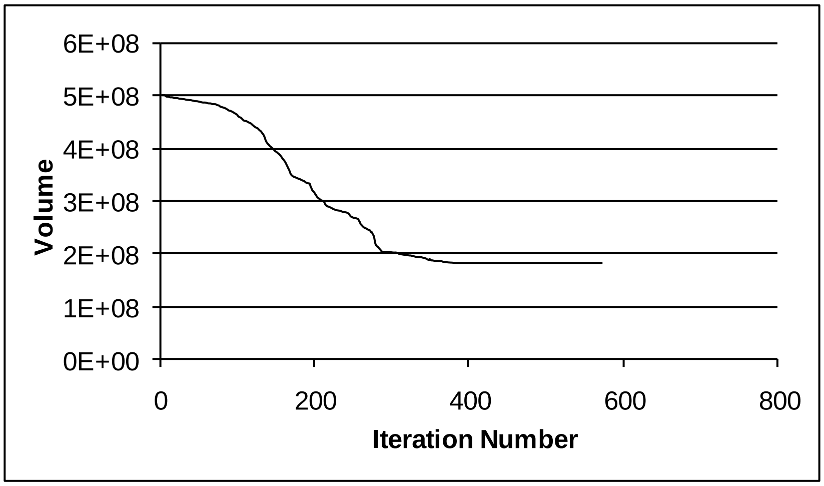


Fig. 26 Design history for the michell beam volume

4. Summary and Conclusions

Evolutionary structural optimization techniques make use of adaptive growth principal that is observed in living-beings in the nature. The basic concept of this principal is simply remove material from the regions that are under stressed and add material to those regions that are overstressed. The recursive application of this concept leads a topology of a structure in which two types of materials are efficiently utilized. It is shown that such algorithms do not deal with number of constraints that are present in the classical optimization approach. The algorithm presented takes into account the realistic feature of composite structural materials having different allowable stress values in tension and compression. The initial threshold stress value is kept quite small so that each iteration small amount of material is removed from the structure. The optimum topologies obtained in the design examples considered are promising.

References

M. Osaki, C.C. Swan, (2002), “Topology and geometry optimization of trusses”, Recent Advances in Optimal Structural Design, Ed. S.A. Burns, Chapter 5, pp: 97-124, ASCE

W.M. Jenkins, “Towards structural optimization via genetic algorithm”, Computers and Structures, Vol. 40, pp: 1321-1327, 1991

B.H.V. Topping, A. I. Khan, J.P. Leite, “Topological design of truss structures using simulated annealing”, Structural Engineering Review, Vol. 8, pp: 301-314, 1996

Y.M. Xie, G.P. Steven, “Evolutionary Structural Optimization”, Springer-Verlag, Berlin, 1997

C. Mattheck, (1997), “Design in nature: Learning from trees”, Springer-Verlag, Berlin

F. Walther, C. Mattheck, (1993), “Local stiffening and sustaining of shell structures by SKO and CAO” Proceedings of International Conference on Structural Optimisation, Zaragoza, Spain, edited by C. A. Brebbia and S. Hernandez, pp:181-188, Computational Mechanics, Southampton, U.K.

E. Hinton, J Sienz, (1995), “Fully stresses topological design of structures using an evolutionary approach”, Engineering computations, 12,229-244

E. Hinton, J Sienz, B. Hassani, (1996), “Fully integrated design optimization for engineering structures with benchmarking”, Advances in Structural Engineering Optimization, Ed. B H V Topping, Civil-comp press, Edinburgh, pp:1-21

Y. M. Xie, G. P. Steven, (1992), “Optimal design of multiple load case structures using an evolutionary procedure”, Internal report, FEARC-9205, Finite Element Analysis Research Center, University of Sydney, Sydney, Australia

Y. M. Xie, G. P. Steven, (1993), “A simple evolutionary procedure for structural optimization”, Computers and Structures, Vol. 49, pp: 885-896

Y. M. Xie, G. P. Steven, (1994), “Optimal design of multiple load case structures using an evolutionary procedure”, Engineering Computations, Vol. 11, pp: 295-302

V. Young, O. M. Querin, G. P. Steven, Y. M. Xie, (1999), “3D and multiple load case bi-directional evolutionary structural optimization”, Structural Optimization, Vol. 18, pp: 183-192

S. SAVAS, M. ULKER, M. P. SAKA, (2003),” Evolutionary Topological Design of Three Dimensional Solid Structures”, Civil and Structural Engineering Computing, B H V Topping, Civil-comp press, Stirling Scotland, pp:132

Van Wyk.D, J. DAVID, (2006) “The Design of Advanced Composite Structures Using Evolutionary Design Methods” ASME 8th Biennial Conference on Engineering Systems Design and Analysis July 4–7, 2006 , Torino, Italy