Randomly Placed Line Source In The Presence Of Perfectly Electromagnetic Conducting Plane

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Abstract

An analytic theory for the electromagnetic scattering from a perfect electromagnetic conducting (PEMC) plane on which a line source has been randomly placed, is developed by using the duality transformation which was introduced by Lindell and Sihvola. The theory allows for the occurrence of cross-polarized fields in the scattered wave, a feature which does not exist in standard scattering theory. This is why the medium is named as PEMC. PEMC medium can be transformed to perfectly electric conducting (PEC) or perfectly magnetic condicting (PMC) media. As an application, plane wave reflection from a planar interface of air and PEMC medium is studied. PEC and PMC are the limiting cases, where there is no cross-polarized component.

1 Introduction

In the early theories of Young, Fresnel, and Kirchhoff, the diffracting obstacle was supposed to be perfectly black, that is to say, all radiations falling on it was assumed to be absorbed and non reflected. This is an inherent source of ambiguity in that such a concept of absolute blackness cant legitimately be defined with precision. It is indeed, incomparable with electromagnetic theory. The problem we are considering, i.e., scattering from half plane, strip or grating are very well known in the field of electromagnetics [1]. Main aim is not to resolve these problems but introduce few random parameters in these planner boundaries for the PEC cases and to study the effects of the stochastic nature of these boundaries on the scattered field. Before to examine the random boundaries, i.e., scatterers with random parameters it is instructive to examine the behavior of randomly placed line source, because in two dimensional planner perfectly conducting boundaries, with sharp edges. An effort has been made to approximate edge diffraction by line source, in far zone. In this paper, the solution for the following average scattered field has been transformed from pec to pemc randomly placed line source.

2 Formulation Of The Problem

The geometry of line source is shown in the Fig.(1),

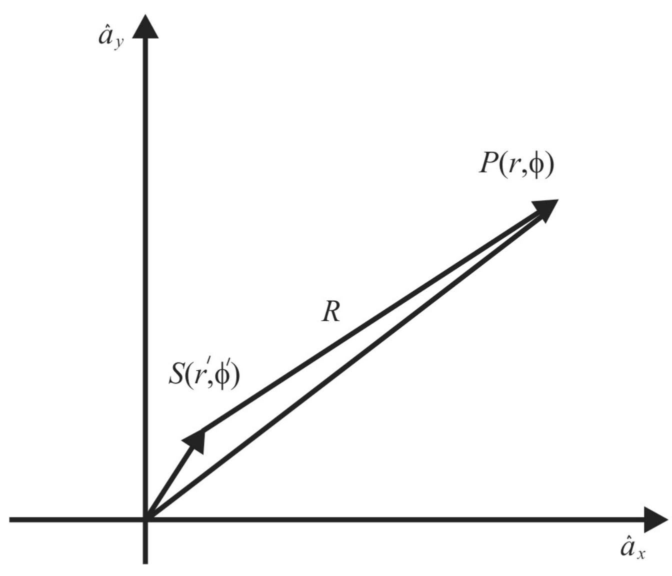

Figure 1: Geometry of the problem

where it has been assumed that its length extends to infinity and electric current is represented by $I = \hat{a}_z I_0$, where I_0 is constant. The electric field at point $P(r,\phi)$ due to this line source can be written as, [2].

$$E = E_z \hat{a}_z = -I_0 \frac{\omega\mu}{4} H_0^1(kR)\hat{a}_z \tag{1}$$

where $R = \sqrt{r^2 + r'^2 - 2rr'\cos(\phi - \phi')}$ and (r',ϕ') is source location. In the present work, the exact location of the randomly placed random width strip is unknown provided its probabilistic knowledge only, i.e, according to statistical theory r' and ϕ' are random variables, and the field $E(r,\phi)$ is a function of these random variables. Due to randomness in source location, the radiated field is also random. Here, main thing is the statistics of this E-field, at least in the first two moments.

Consider the case when only r' is random with certain probability density function $P_{r'}(r')$ and ϕ' is deterministic. E_z in the far zone $(r >> r')$ can be written as,

$$E_z = -I_0 \frac{\omega\mu}{4} \sqrt{\frac{2}{i\Pi}} \frac{e^{ikR}}{\sqrt{kR}} \tag{2}$$

where $R; r - r'\cos\psi$ and $\psi = \phi - \phi'$.
Further, above expression can be shown Mathematically as

$$E_z = -I_0 \frac{\omega\mu}{4} \sqrt{\frac{2}{i\pi}} \frac{e^{ikr}}{\sqrt{kr}} e^{-ikr'\cos\psi} \tag{3}$$

Here r' is random variable with probability density function $P_{r'}(r')$. Making use of statistics of source location,the average E-field can be calculated as

$$< E_z >= -I_0 \frac{\omega\mu}{4} \sqrt{\frac{2}{i\pi}} \frac{e^{ikr}}{\sqrt{kr}} < e^{-ikr'\cos\psi} > \tag{4}$$

where

$$< e^{-ikr'\cos\psi} >= \int_{-\infty}^{\infty} e^{-ikr'\cos\psi} P_{r'}(r')dr' \tag{5}$$

The second moment of this field will be

$$< |E_z|^2 >=< E_z E_z^* >; I_0^2 \frac{(\omega\mu)^2}{8\pi} \frac{1}{kr} \tag{6}$$

and the variance of the field can be evaluated as

$$var(E_z) =< |E_z|^2 > -(< E_z >)^2. \tag{7}$$

Consider r' be exponentially distributed random variable with probability density function given by

$$P_{r'}(r') = \lambda e^{-\lambda r'}, r' \geq 0 \tag{8}$$

where $\lambda = 1/r'$ and $< r' >$ is the average value of r'. Using the probability density function

$$< e^{-ikr'\cos\psi} >= \lambda \int_0^{\infty} e^{-ikr'\cos\psi} = \frac{1}{1+ik<r'\cos\psi>} \tag{9}$$

Now using this result in above equation, the average field can be written as

$$< E_z >= -I_0 \frac{\omega\mu}{4} \sqrt{\frac{2}{i\pi}} \frac{e^{ikr}}{\sqrt{kr}} \frac{1}{1+ik<r'>cos\psi} \tag{10}$$

Taking the modulus square of the above equation, the expression is given below.

$$| < E_z > |^2 = I_0^2 \frac{(\omega\mu)^2}{8} \frac{1}{\pi} \frac{e^{ikr}}{\sqrt{kr}} \frac{1}{1+k^2<r'>\cos^2\psi} \tag{11}$$

and the variance of E_z field can be written as

$$Var(E_z) = I_0^2 \frac{(\omega\mu)^2}{8} \frac{1}{\pi} \frac{e^{ikr}}{\sqrt{kr}} \frac{k^2<r'>\cos^2\psi}{1+k^2<r'>\cos^2\psi} \tag{12}$$

Now consider the case when both r' and ϕ' are random. We assume that when both r' and ϕ' are staistically independent random variable. Also assume that ϕ' is uniformly distributed,i.e. probability density function of ϕ' is given by,

$$P_{\phi'}(\phi') = \frac{1}{2\pi}, 0 \leq \phi' < 2\pi \tag{13}$$

Due to a statistical independence assumption the joint probability density function of r' and ϕ' will be

$$P_{\phi' r'}(\phi', r') = P_{r'}(r')P_{\phi'}(\phi') \tag{14}$$

By calculating the average field as

$$< E_z >= -I_0 \frac{\omega\mu}{4}\sqrt{\frac{2}{i\pi}}\frac{e^{ikr}}{\sqrt{kr}} < e^{-ikr'\cos(\phi-\phi')} > \tag{15}$$

In this case $< e^{-ikr'\cos\psi} >$ will become

$$< e^{ikr'\cos\psi} >= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P_{\phi' r'}(\phi', r') e^{-ikr'\cos(\phi-\phi')} dr' d\phi' \tag{16}$$

$$< e^{-ikr'\cos\psi} >= \int_{-\infty}^{\infty} P_{r'}(r') \int_{-\infty}^{\infty} e^{-ikr'\cos(\phi-\phi')} d\phi' dr' \tag{17}$$

By using the Bessel Function identity

$$J_0(a) = \frac{1}{2\pi}\int_0^{2\pi} e^{-ia\cos(\phi-\phi')} d\phi', \tag{18}$$

and the result

$$\int_0^{\infty} J_0(kr') e^{-\lambda r'} dr' = \frac{1}{\sqrt{k^2+\lambda^2}} \tag{19}$$

we can write

$$< e^{-ikr'\cos\psi} >= \frac{\lambda}{\sqrt{k^2+\lambda^2}} = \frac{1}{\sqrt{1+k^2<r'>^2}}. \tag{20}$$

Hence the avaerage scattered field can be written as

$$< E_z >; -I_0 \frac{\omega\mu}{4}\sqrt{\frac{2}{i\pi}}\frac{e^{ikr}}{\sqrt{kr}}\frac{1}{\sqrt{1+k^2<r'>^2}} \tag{21}$$

and the variance can be calculated as,

$$var(E_z) = I_0^2 \frac{(\omega\mu)^2}{8\pi}\frac{1}{kr}\left(\frac{k^2<r'>^2}{1+k^2<r'>^2}\right) \tag{22}$$

It is observed that average field and its variance are independent of ϕ. The above average scattered field can be transformed from perfectly electric conducting case to perfectly electromagnetic conducting case by the following theory. The Concept of PEMC introduced by Lindell and Sihvola [3, 4] is a generalization of both PEC and PMC. An analytic theory for the electromagnetic scattering from a PEMC plane where a line source has been placed randomly, is developed. The PEMC medium characterized by a single scalar parameter M, which is the admittance of the surface interface, where $M = 0$ reduces the PMC case and the limit $M \to \pm\infty$ corresponds to the perfect electric conductor (PEC) case. The theory allows for the occurrence of cross-polarized fields in the scattered wave in the scattered wave, a feature which does not exist in standard scattering theory. This means that PEC and PMC are the limiting cases, for which there is no cross-polarized component. Because the PEMC medium does not allow electromagnetic energy to enter, an interface of such a medium behaves as an ideal boundary to the electromagnetic field. At the surface of a PEMC media, the boundary conditions between PEMC medium and air with unit normal vector n, are of the more general form. Because tangential components of the E and H fields are continuous at any interface of two media, one of the boundary conditions for the medium in the air side is $n \times (H + ME) = 0$, because a similar term vanishes in the PEMC-medium side. The other condition is based on the continuity of the normal component of the D and B fields which gives another boundary condition as $n.(D - MB) = 0$.
Here, PEC boundary may be defined by the conditions

$$n \times E = 0, \qquad n.B = 0 \tag{23}$$

While PMC boundary may be defined by the boundary conditions

$$n \times H = 0, \qquad n.D = 0 \tag{24}$$

where M denotes the admittance of the boundary which is characterizes the PEMC. For $M = 0$, the PMC case is retrieved, while the limit $M \to \pm\infty$ corresponds to the PEC case. Possibilities for the realization of a PEMC boundary have also been studied [5].

It has been observed theoretically that a PEMC material acts as a perfect reflector of electromagnetic waves, but differs from the PEC andthe PMC in that the reflected wave has a cross-polarized component. The duality transformations of perfectly electric condutor (PEC) to PEMC have been studied by many researchers [3, 4, 5, 6, 7, 8, 9]. Here we present an analytic scattering theory for a PEMC step, which is a generalization of the classical scattering theory.

Applying a duality transformation which is known to transform a set of fields and sources to another set and the medium to another one. In its most general form, the duality transformation can be defined as a linear relation between the electromagnetic fields. The effect of the duality transformation can be written by the following special choice of transformation parameters:

$$\begin{pmatrix} E_d \\ H_d \end{pmatrix} = \begin{pmatrix} M\eta_0 & \eta_0 \\ \frac{-1}{\eta_0} & M\eta_0 \end{pmatrix} \begin{pmatrix} E \\ H \end{pmatrix} \tag{25}$$

has the property of transforming PEMC to PEC, while

$$\begin{pmatrix} E \\ H \end{pmatrix} = \frac{1}{(M\eta_0)^2+1} \begin{pmatrix} M\eta_0 & -\eta_0 \\ \frac{1}{\eta_0} & M\eta_0 \end{pmatrix} \begin{pmatrix} E_d \\ H_d \end{pmatrix} \tag{26}$$

has the property of transforming PEC to PEMC [4].

Following the above relations [3], the transformed equations becomes as

$$E^r = -\frac{1}{M^2\eta_0^2+1}\left[(-1+M^2\eta_0^2)E^i + 2M\eta_0 u_z \times E^i\right] \tag{27}$$

$$E_{sd} = -(M\eta_0 E_s + \eta_0 H_s) \tag{28}$$

$$H_{sd} = -\frac{1}{\eta_0}E_s + M\eta_0 H_s \tag{29}$$

$$E_s = \frac{1}{(M\eta_0)^2+1}[M\eta_0 E_{sd} - \eta_0 H_{sd}] \tag{30}$$

$$E_s = \frac{1}{(M\eta_0)^2+1}[((M\eta_0)^2-1)E_s - 2M\eta_0^2 H_s] \tag{31}$$

$$E_s = \frac{1}{(M\eta_0)^2+1}[((M\eta_0)^2-1)E_s - 2M\eta_0 E_s] \tag{32}$$

Where E_s, H_s are transformed pemc average fields and E_{sd}, H_{sd} are average scattered elecric and magnetic fields respectively.

This means that, for a linearly polarized incident field E^i, the reflected field from a such a boundary has a both co-polarized component, while $u_z \times E^i$ is a cross-polarized component, in the general case. For the PMC and PEC special cases ($M = 0$ and $M = \pm\infty$ respectively), the cross-polarized component vanishes. For the special PEMC case $M = \frac{1}{\eta_0}$, such that

$$(E^r = -u_z \times E^i) \tag{33}$$

which means that the reflected field appears totally cross-polarized. It is obvious theoretically that a PEMC material acts as a perfect reflector of electromagnetic waves, but differs from the PEC ($E^r + E^i) = 0$ and $H^r = H^i$) and PMC ($E^r = E^i$ and $H^r + H^i = 0$) in that the reflected wave has a cross-polarized component.

3 Concluding remarks

In this work, a plane wave scattering by a perfect electromagnetic conducting plane on which a line source has been randomly placed, has been studied.

The theory provides explicit analytical formulas for the electric and magnetic field. An other formulla has been derived for the relative contributions to the scattered fields of the co-polarized and the crosspolarized fields depend on parameter M. The cross-polarized scattered fields vanish in the PEC and PMC cases, and are maximal for $M = \pm 1$. In the general case,the reflected wave has both a co-polarized and a cross-polarized component. The above transformed solution presents an analytical theory for the scattering of a perfect electromagnetic plane for a randomly placed line source. It is clear from the above discussion that for $M \to \infty$ and $M \to 0$ correspond to the PEC and PMC respectively. Moreover, for $M = \pm 1$ the medium reduces to PEMC.

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