Calculation of System Critical Time T_{po} by Applying Modified CPM Method

Ján Tvarožek
Faculty of special technology
Alexander Dubček University of Trenčin
Slovakia

Július Alcnauer
Faculty of management
University of Prešov
Slovakia

Introduction

System reliability in broad sense, is its ability to complete its main tasks under given circumstances of exploitation. In connection with this characteristic of system is constantly present one of fundamental questions, to which is necessary to know the answer:

„What is the degree of certainty that in time t_o will be specific system S ready to fulfill, respectively, will fulfill given task?“

Probability P ($\Delta\tau$), that system will fulfill its core tasks in given tactical and technical conditions over a period of time, (we call it time of urgent operational needs - $\Delta\tau$) assuming that system is at the beginning of that time without failure, is equal to:

$$P(\Delta\tau) = P_\tau(\Delta\tau)\cdot P_\tau(\Delta\tau)$$

P ($\Delta\tau$) – technical reliability of system, [1],
P_τ ($\Delta\tau$) – functional reliability of system, i.e. likelihood of completing main tasks by system. (Quality of system „work“, which is determined as a general rule of system work precision) [3].

Functional reliability can be written down to following form:

$$P_\tau(\Delta\tau) = P_{F1}(\Delta\tau_1)\cdot P_{F2}(\Delta\tau_2)\cdot P_{\tau 3}(\Delta\tau_3)......P_{FK}(\Delta\tau_K)$$

probability of successful conversion

$P_{FK}(\Delta_{\tau K})$ – series of successive operations (activities), which ensures total value P_F.

Each system can fulfill the task without outside interference, or noise.

We agreed, that under the term "interference" is understand any organized and unorganized activity that interferes with system during fullfiling its tasks. In that case, functional reliability will be expressed as follows:

$$P_F = P_{oF}\cdot P_R + P_{RF}(1-P_R)$$

P_{oF} – likelihood of fulfiling of tasks by system without breaching,
P_R – probability that during time of urgent operational needs, breach will not be created,
P_{RF} – likelihood of fulfiling of tasks by system during interferention.

Functional reliability depends greatly on recoverability, maintainability, and other, often randomly acting factors. Therefore, giving complete and accurate answer to initial question is not currently possible for systems designed for combat use. For purpose of simplicity, therefore, we restrict our considerations to issues of technical preparatin of system for use. Whilst neglecting influence of other factors on process of system preparation, i.e. we will assume that technical preparation for use and application of system take place in a relatively isolated system, which is unaffected by any external interference signals. (However, system has managed inputs).

Formulation of the problem we have to solve, then, will be as follows:

To determine when, with which time advance Δτ, to start system S preparation to fulfillment of task, so that at time t, is achieved desired level of operational reliability Y.

Definition: In practice is often considered sufficient to determine time Δτ by simply adding up technologically required time intervals, to which is added some value, usually determined by practical experience. This value is of course considerably subjective.

We solve this problem by using methods of operational research.

In order to shied away from misunderstanding, I will have to start with a few definitions [1]:

System and its elements may be present only in two states:

- capable of operating,
- incapable of operating.

System is considered unserviceable only in the moment when only one of its basic parameters exceeds (falls below) value of permissible exploitative parameter.

System failure is its incapability of operating under given circumstances. Removal of this condition lasts less or same time as predetermined time, so called "critical" system time – T_{po}.

System crash is its incapability of operating under given circumstances, but removal of this condition takes longer than critical time T_{po}.

Military system is considered to be able to fight in a period of time, when under given tactical and technical conditions doesn´t have any problems, system is operationally reliable, if at the beginning of interval of urgent operational need Δτ is able to fight and during this interval, won´t arise its crash. We evaluate degree of operational reliability Y(t, θ) by using likelihood of crash H (t, θ):

$$Y(t,\theta) = 1 - H(t,\theta) \tag{1}$$

From literature (e.g. [5].) is sufficiently well known methodology for evaluation of system in terms of its technical reliability, using the function - likelihood of trouble-free operation. For period of normal exploitation applies:

$$P(\Delta t) = P_{\Delta} \cdot \exp(-\Delta \cdot \Delta t) \tag{2}$$

P_{Δ} – initial reliability of system at the beginning of time interval Δt,
Δ – intensity of system failures.

Moreover, parameter of operational reliability of system – availibility factor K_h is well known:

$$K_h = \frac{\overline{T}}{\overline{T} + \overline{T_o}} \tag{3}$$

$\overline{T}$ - mean time between system failures,
$\overline{T_o}$ - mean time of system recovery.

Why not introduce some other parameter by which we evaluate system combat readiness - operational reliability Y?

Any operation is in progress at time. Threat of high combat readiness of system is not only the fact that a failure occurs, but also length of follow-forced system downtime – T_P during which we are trying to restore operability. Therefore we distinguish failure from crash, depending on how long will take failure removal, i.e. how long will run system recovery.[2]. Length of downtime will depend on system recoverability and in a large extent from whole system recovery. Causes of crashes can be different (lack of spare parts, low degree of recoverability, poorly trained operator and so on.), But result is always the same - combat task will not be fulfilled. Crash is random phenomenon, which is necessary to count with, even assuming that we have done everything in order to the most perfect quality of operating system. System availibility factor does not express these facts as clearly as degree of system operational reliability.

Probability of crash: Let have some system S, which is prepared to use. Set of consecutive, eventually parallel or otherwise mutually ongoing operations associated with this preparation can easily be expressed in edge evaluated network graph – for example by CPM method. Network critical path in which operations are evaluated by technologically necessary time for system preparation – i.e. expresses maximum preparation time without considering failure occurence and subsequent loss of time used for system recovery.

It is certainly reasonable to assume that the whole process of preparation will be done "according to plan" only provided that during this preparation anywhere, at any stage of system preparation, a crash won´t arise. This means that probability of system preparation without a crash can be described by relation:

$$Y(t,\theta) = \coprod_{i,j}^{k,1} Y_{i,j}\left(t_i, j : \theta_{i,j}\right) \tag{4}$$

where: i,j and k.l are operations lying on critical path,

$\theta_{i,j}$ is system recoverability at stage < i, j >,

$t_{i,j}$ time of system preparation at stage < i, j > given by preparation technology.

On the other hand, it is unlikely that during system preparation somewhere, at some stage of preparation, did not occur a failure. Possibility of system crash, however, arises whenever a failure occurs. In fact, only a portion of failures is amended to crashes, and that because each system has certain operating system, located in immediate vicinity of system that performs recovery. Qualities of this recovery system are primarily given due to possibility to carry 1, 2, 3 ..... N recoveries. For obvious reasons, likelihood of this system is limited. (Operating personnel, equipment, backup elements supplies, etc..). Particularly, there is little possibility to carry several identical system recoveries.

System recoverability - θ is evaluated by likehood of system recovery in time less or equal to t. Then applies:

$$\theta(t) = \int_o^t f(T_o)\, dT_o \tag{5}$$

where f (T_o) is probability density of random variable T system recovery time.

We will call probability that system will not be recovered over time T_{po}, operational unreliability factor K_p.

$$K_p = 1 - \theta(T_{po}) = 1 - \int_o^{T_{po}} f(T_o)\, dT_o \tag{6}$$

Probability of that crash won't occur over time Δτ, e.g. system is operationally reliable, will be equal to:

$$Y(\Delta\tau.\theta) = P_o(\Delta\tau) + P_1(\Delta\tau)\cdot\theta_1(T_{po}) + P_2(\Delta\tau)\cdot\theta_2(T_{po}).... + P_N(\Delta\tau)\cdot\theta_N(T_{po}) \tag{7}$$

where θ_N (T_{po}) is probability of system recovery with occurence of maximum N failures over time less than or equal to critical time T_{po},

P_N (Δτ) is likelihood of N failures occurence over time Δτ,

P_o (Δτ) = P (Δτ) is likelihood of fault-free system operation,

$$\lim_{N=\infty} \sum_{i=1}^{N} P_i(\Delta\tau) = 1 - P(\Delta\tau) = Q(\Delta\tau)$$

is likelihood of at least one failure.

Likelihood of one or more failures in reliable systems is very small. Therefore, we can accept

$$Q(\Delta\tau) = \sum_{i=1}^{N} P_i(\Delta\tau)$$

In that case, we can organize system so, that probability of recovery over critical time T_{po} with great precision is equal to:

$$\theta_N(T_{po}) = \theta(T_{po}).$$

Now we will modify equation (7) like this:

$$Y(\Delta\tau,\theta) = P(\Delta\tau) + Q(\Delta\tau)\cdot\theta(T_{po}) \qquad (8)$$

Likelihood of system crash will be then

$$H(\Delta\tau,\theta) = 1 - \lfloor P(\Delta\tau)\cdot\theta(T_{po})\rfloor \qquad (9)$$

We can determine functions P (Δτ) and Q (Δτ) using theory of reliability. Therefore, it is especially when determining operational reliability or calculating value θ (T_{po}).

Probability density of system recovery f (T_o) is usually expressed using the Erlang distribution:

$$f_N(T_o) = \left(\frac{N}{\overline{\overline{T_o}}}\right)^N \cdot \frac{T_o^{N-1}}{I'(N)} \cdot \exp\left(-\frac{NT_o}{\overline{\overline{T_o}}}\right) \qquad (10)$$

where N is degree of Erlang distribution, which in this case means that recovery system will perform N-1 recoveries, but N-th recovery can´t be performed. (For example, there is not enought spare elements etc.).

Suppose we have system S, which is operated by cascade characterized by possibility to perform only one recovery.
Then:

$$f_2(T_o) = \frac{4}{\overline{\overline{T_o^2}}}\cdot T_o \cdot \exp\left(-\frac{2T_o}{\overline{\overline{T_o}}}\right),$$

$$\theta_2(T_{po}) = \int_o^{T_{po}} f_2(T_o)dT = 1 - (1+2\beta)e^{-2\beta} \qquad (11)$$

where $T_{po} = \beta\overline{T_o}$ *and* β are dimensionless coefficient. (12)

In case that we have more perfect cascade of system recovery, which is characterized by Erlang distribution of 3rd grade (N = 3), probability density of system recovery will be:

$$f_S(T_o) = \frac{27}{2\cdot\overline{\overline{T_o^2}}}\cdot T_o^2 \cdot \exp\left(-\frac{3T_o}{T_o}\right),$$

$$\theta_3(T_{po}) = \int_0^{T_{po}} f_3(T_o)dT_o = 1 - (1+3\beta+4{,}5\beta^2)e^{-3\beta} \qquad (13)$$

Values θ_N and K_p in dependence on coefficient β are shown in Table 1.

Calculation of critical time T_{po}:

From analysis of equations (11) and (13) is clear that by increasing coefficient β, we can achieve arbitrarily large values of likelihood of restoration θ (T_{po}) and thus reduction of operational reliability coefficient K_p. At the same time, this means that critical time T_{po} is excessively prolonged, which in turn leads to raising crash likelihood (9).

Table 1 Dependence of the probability of recovery θ_N and operational reliability coefficient k_P on coefficient β.

β	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0	1,2
θ_2	0,01756	0,06158	0,12192	0,19126	0,26420	0,33736	0,40816	0,45706	0,53716	0,59410	0,69155
K_{p2}	0,98244	0,93842	0,87808	0,80874	0,73580	0,66264	0,59184	0,52494	0,46284	0,40590	0,30845
θ_3	0,00362	0,02292	0,06286	0,12053	0,19115	0,26937	0,35035	0,43029	0,50637	0,57679	0,69725
K_{p3}	0,99638	0,97708	0,93714	0,87947	0,80885	0,73063	0,64965	0,56971	0,49363	0,42321	0,30275
β	1,4	1,6	1,8	2,0	2,2	2,4	2,6	2,8	3,0	3,5	4,0
θ_2	0,76892	0,82881	0,87433	0,90840	0,93369	0,95227	0,96578	0,97558	0,98625	0,99270	0,99699
K_{p2}	0,23108	0,17119	0,12567	0,09160	0,06631	0,04773	0,03422	0,02442	0,01735	0,00730	0,00301
θ_3	0,78976	0,85746	0,90524	0,93803	0,95935	0,97453	0,98388	0,98984	0,99377	0,998	0,9999
K_{p3}	0,21024	0,14254	0,09476	0,06197	0,04065	0,2547	0,01612	0,01018	0,00623	0,002	0,0001

Interval of urgent operational need for each stage of preparation $\Delta\tau_{ü}$ consists of two time slots:

- from time $t_{ü}$ required to perform operations given by preparation technology without using means,
- from time backup in case that, there would be a need to perform system recovery, i.e. critical time for given stage of preparation T_{poi}.

The longer the interval $\Delta\tau_{ü}$ the greater is possibility of failure. Critical time T_{poi} for every stage of system preparation for use, is given by mean recovery time T_{oi} at this stage and coefficient β_i. Consideration that at each stage of preparation will occur a failure won´t make any sense. Likelihood of such phenomenon is small [1:4]. Therefore, when determining time T_{po} is reasonable to consider only part of critical time T_{poi} for each stage of system preparation for use. Criterion for this choice may be failure probability Q (t) and level of system operational reliability Y_i (t, θ) at given stage of system preparation for use.

Procedure for calculating critical system time – T_{po} then could be this:

1. We will illustrate whole process of system preparation by edge evaluated network graph, where values of each edge represent information $t_{ü}$.

2. Using CPM method, we can find "additional critical path" in the network and mark it with moderately strong line (colour). Number of activities lying on critical path is marked as computational information „K". Thus evaluated network informs us about time required to system preparation only in terms of technology itself, without thinking of failures. We determine probability of fault-free operation P ($\Delta\tau_{ü}$) and probability of failure Q ($\Delta\tau_{ü}$) = 1 – P ($\Delta\tau_{ü}$) for each stage of preparation. While, for the needs of calculation, we consider $\Delta\tau_{ü} \cong t_{ü}$.

3. Based on required level of operational reliability Y, we determine required level of operational reliability at each stage of preparation Y_i (4). In order of simplicity, we consider the same value Y_i at each stage.

$$\ln Y_i = \frac{\ln Y}{"K"}.$$

4. According to level of operational reliability Y_i and value of probability of fault-free operation P ($t_{ü}$) we determine required degree of recoverability $\theta_{ü}$ due to system recovery patency approximately expressed by degree of Erlang distribution at given stage N_i.

$$\theta_{ü} = \frac{Y_1 - P(t_{ü})}{1 - P(t_{ü})} \qquad for\ Y_i \geq P(t_{ü}),$$

$$\theta_{ü} = 0 \qquad for\ Y_i \lhd P(t_{ü}).$$

5. We determine coefficient βi for each stage of preparation < i, j > from Table 1.

6. We determine critical time T_{poi} for each stage:

$$T_{poi} = \beta_i \cdot \overline{T_{oi}}.$$

7. We will calculate interval of urgent operational need for given stage of preparation: $\Delta\tau_{ü} = t_{ü} + T_{poi}$.

8. Thus calculated time $\Delta\tau_{ü}$ is written into netvork graph to edge of corresponding operation and put it into the box. Using CPM we will find „main critical path" in network only on the basis of $\Delta\tau_{ü}$ values. We will mark main critical path in network by strong line (colour). Network evaluated in this way expresses time needed to system preparation for use as for technological point of view, as well as for failure rate.

9. Total system preparation time (e.g. urgent operational need interval) for use $\Delta\tau$ is then equal to amount of times $\Delta\tau_{ü}$ on main critical path and critical time T_{po} is equal to amount of times T_{poi} on main critical path. In doing so, however, is ensured (in terms of recoverability) previously required level of operational reliability Y.

Example: Suppose, that is necessary to prepare operational-tactical missile unit for use, so that it is ready for combat use in time t_o = 06,00 hrs. on day X, desired operational reliability is Y = 0,8.

Let progress of preparation of missile unit be network, illustrated in Fig. 1 and its edge evaluation is in accordance with calculation method described above. All times are listed in hours.

We show the entire calculation in Table 2. Strongly printed data (color) are given, calculated values are printed slightly (different color).

We use Table 3 for simplification of calculation.

Table 2: The calculation of time periods (t) for the department of operational-tactical missiles at the required operational reliability (Y)

Y = 0,8			„K“ = 5		$\ln Y_1 = \frac{\ln Y}{"K"} = \frac{9{,}90309}{5} - 2$					Y_i = 0,9564			note
operation <i, j >	τ_{ij}	$\overline{T_{ij}}$	Δ_{ij}	Δ_{ij}-τ_{ij}	P (τ_{ij})	1-P(Δ_{ij})	Θ_{ij}	N_i	β_i	$\overline{T_{oi}}$	T_{poi}	$\Delta\tau_{ij}$	
0-1	1	10	0,1	0,1	0,90484	0,09516	0,543	2	0,9	0,1	0,09	1,09	
1-2	2	4	0,25	0,5	0,60653	0,39347	0,894	3	1,8	0,01	0,018	2,02	
1-3	2	10	0,1	0,2	0,81873	0,18127	0,76	2	1,4	0,2	0,28	2,28	
2-4	2	10	0,1	0,2	0,81873	0,18127	0,76	2	1,4	0,2	0,28	2,28	
2-5	3	10	0,1	0,3	0,74082	0,25918	0,828	2	1,6	0,5	0,8	3,8	
3-4	3	10	0,1	0,3	0,74082	0,25918	0,828	2	1,6	0,2	0,32	3,32	$\Delta\tau$ = 14,67
3-6	5	10	0,1	0,5	0,60653	0,39347	0,894	3	1,8	0,5	0,9	5,9	
4-7	4	100	0,01	0,004	0,96079	0,03921	0	2	0	1	0	4	
5-7	3	5	0,2	0,6	0,54881	0,45119	0,905	3	1,8	2	0,36	6,6	
6-7	1	5	0,2	0,2	0,81873	0,18127	0,76	3	1,4	0,1	0,14	1,14	
7-8	1	3	0,33	0,33	0,71600	0,28400	0,845	2	1,6	0,1	0,16	1,16	

Table 3: Table for determining required recoverability $\theta_{ü}$ depending on system reliability $P_{(tij)}$ and required system operational reliability Y

P/Y	0,9999	0,9995	0,9990	0,995	0,99	0,95	0,9	0,85	0,8	0,7	0,6	0,4	0,2
0,909	0	9,5	0	0	0	0	0	0	0	0	0	0	0
0,995	0,98	0,9	0,8	0	0	0	0	0	0	0	0	0	0
0,99	0,99	0,95	0,9	0,5	0	0	0	0	0	0	0	0	0
0,95	0,998	0,99	0,98	0,9	0,8	0	0	0	0	0	0	0	0
0,9	0,999	0,995	0,99	0,95	0,9	0,5	0	0	0	0	0	0	0
0,85	0,993	0,996	0,993	0,96	0,93	0,66	0,33	0	0	0	0	0	0
0,8	0,9995	0,9975	0,995	0,975	0,95	0,75	0,5	0,25	0	0	0	0	0
0,7	0,9997	0,9983	0,9976	0,9833	0,9667	0,8333	0,6667	0,5	0,3333	0	0	0	0
0,6	0,9997	0,9987	0,9975	0,9875	0,975	0,875	0,75	0,625	0,5	0,25	0	0	0
0,4	0,9998	0,9992	0,9983	0,9917	0,9833	0,9166	0,8333	0,75	0,6667	0,5	0,3333	0	0
0,2	0,9999	0,9994	0,9987	0,9937	0,9875	0,9375	0,875	0,8125	0,75	0,625	0,5	0,25	0

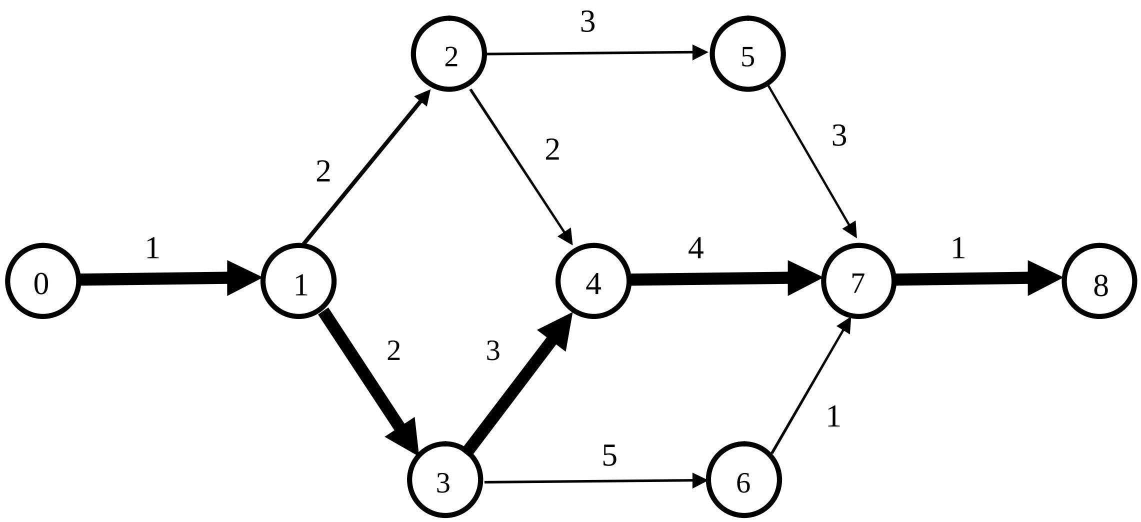

Fig. 1: Network of operational reliability on the progress of preparation of rocket body

From the result of calculation (Fig. 2) is clear, that when considering only preparation technology is time necessary for system preparation equal to 11 hours, whereas when considering system failure rate, with required level of operational reliability Y = 0,8 time is equal to 14 hours a 40 minutes. This means, that we must start preparation of given system at 15,20 hrs. on day X-1. In addition, using this calculation, we find that the most dangerous stages of missile unit preparation for use, in terms of crash likelihood, are operations < 5-7>.

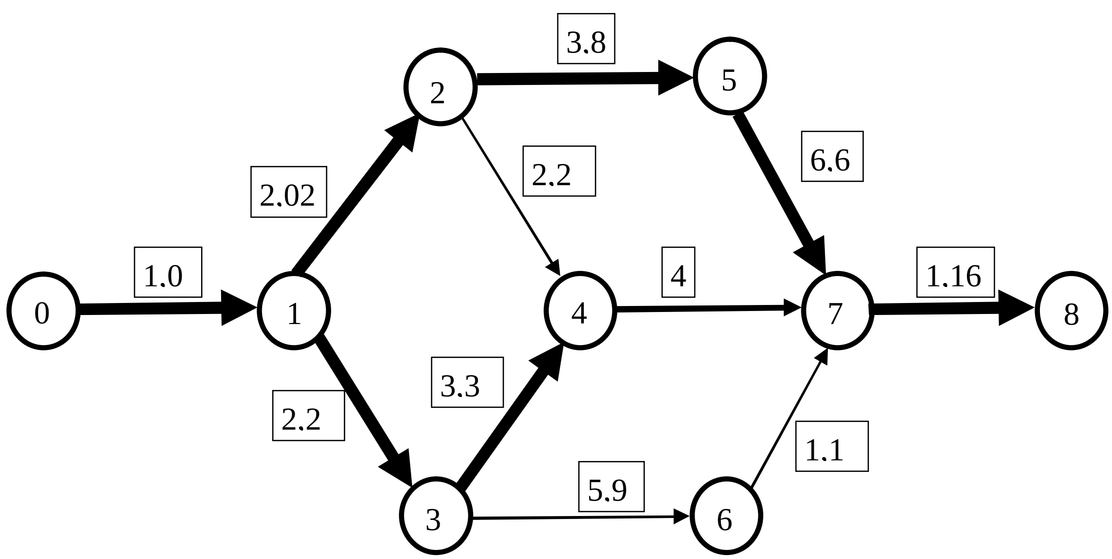

Fig. 2: Calculation of the time required for preparation of missile unit operational reliability

Conclusion

Based on mathematical apparatus of probability theory, authors point out contradiction between reducing system unreliability and the effort to shorten time of system preparation for use. The result of considerations is mentioned methodology for calculating critical system time – T_{po}, which is sufficiently general, and therefore can be used for any system. Some simplifications that are considered, have no significant effect on accuracy of calculation. Use of solution can be expressed using failure rate Δ [1]. Details for calculating, at the present time can be obtained when evaluating data about operation of military technics, for example by means of "Collection and evaluation of data about the operation of missile technics and artillery units", which is tested in selected units.

Use of such system parameters as operational reliability, likelihood of crash, operational unreliability factor, allow to carry calculations of other tasks operational and technical nature, especially - determine the extent of reserves and their dislocation.

Finally, it should be added that authors are aware of limitations of management of given issue by using deterministic model. Nature of the whole process of system preparation for use is strictly probabilistic, and therefore using CPM method is only first step to solve this task by using alternative stochastic network.

Literature:

Kubík, M.: *Teoretické základy exploatácie raketových komplexov.* I. časť, 1966: skriptaVA AZ, por. č. tisku S-275

Kubík, M.: *Úvod do teórie obnovy.* Prednáška VA AZ, K 215, 1966

Lykov, I. A.: *Osnovy nadežnosti radioelektronnoj apparatury.* ARTA, 1963

Saaty Thomas L.: *Mathematical methods of operations research*. Ruský preklad, Moskva, 1983

šišonok, n. a. a ostatní: Osnovy teorií nadežnosti radioelektronnoj techniki. Sov. radio, Moskva, 1984