The Absolute Node Coordinates Method of Large Deformation of the Flexible Beam System

Jing Guo

Chunhua Zhao

College of Mchanical Engineering
Shanghai University of Engineering Science
Shanghai, China

Abstract

The application of an absolute nodal coordinate formulation (ANCF) in the coupling dynamics of flexible beams with large deformation was investigated. By using geometric nonlinear formulation and finite element method, the dynamics equations of planar beams system were developed by using the ANCF. The comparison between the ANC formulation and nonlinear strain-displacement formulation in the case of small deformation and large deformation indicates that the ANC formulation is more precise and suited for the beams with large deformation. The present results are verified by using the energy conservation law.

Keywords: Planar Beams System; Kinetics Equations; Absolute Nodal Coordinate Formulation; Large Deformation

As new materials and new technology application in the engineering, manifest the flexible deformation of motion mechanism, the motion mechanism for a wide range of motion or the applied load, large deformation phenomenon in a flexible body, manifests the geometrical nonlinear behavior. For example, dozens of meters long robotic arm on the international space station and large deformation problems of large area solar panels. In recent years, some scholars to work on a wide range of motion of geometrical nonlinear problem of a cantilever beam was studied [1, 2], Based on the assumption of small deformation, and established an approximation of the coupled dynamic equations, however, due to ignore the higher order term in the dynamics equation, unable to solve the problem of large deformation the coupled dynamics of a cantilever beam. In order to establish a precise dynamic model of Shabana [3] proposed the absolute coordinate method, the characteristics of this method is an absolute displacement expression is simpler, quality array for constant array, simplifies the system model to a certain extent. And, more importantly, in the process of establishing dynamics equations, based on the geometric nonlinear theory, retained the longitudinal strain and elastic force of higher order term. Then, Shabana [4 ~ 6] the absolute coordinates method is applied to large rotation and large deformation of a cantilever beam. In this paper, the absolute coordinates method to plane beam system, starting from the exact expression of axial strain and curvature, the establishment of a flexible beam finite element discrete dynamics equation, on this basis, according to the constraint relation between flexible body movement, the establishment of a flexible beam system dynamics equation. Numerical simulation of flexible pendulum and double pendulum and demonstrate the validity of the modeling theory of absolute node coordinates.

1. The Dynamic Equation of Single Beam

In this paper, the research object of plane beam system, using Euler Bernoulli assumption, without considering shear deformation, that the deformation of beam cross section remains for plane and vertical axis. Establish Ox_0y_0 absolute coordinate system, as shown in figure 1.For plane beam system, including the I beam, the beam is divided into several units, $O_ex_ey_e$ is consolidation in the beam end unit of floating coordinate system.

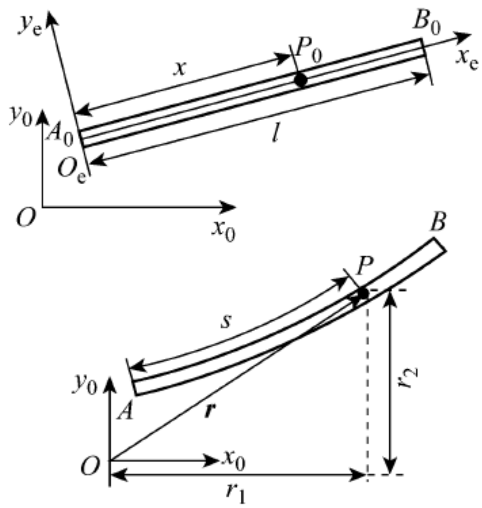

Fig 1: Beam Element before and After Deformation

As shown in figure 1, r for arbitrary point on the beam axis P about absolute coordinates the coordinates of the matrix, can be expressed as：

$$r = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = Se \qquad (1)$$

r_1, r_2, respectively is the absolute displacement in x_0, y_0 direction; S is the shape function; E array as a unit node coordinates.

$$e = \begin{bmatrix} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \end{bmatrix}^{\mathrm{T}}$$

Absolute displacement of node:

$$e_1 = r_1\big|_{x=0}, \quad e_2 = r_2\big|_{x=0}$$

$$e_5 = r_1\big|_{x=l}, \quad e_6 = r_2\big|_{x=l}$$

$$s_1 = 1 - 3\mathrm{Y}^2, \quad s_2 = \mathrm{Y} - 2\mathrm{Y}^2 + \mathrm{Y}^3$$

$$s_3 = 3\mathrm{Y}^2 - 2\mathrm{Y}^3, \quad \mathrm{Y} = x/l$$

The kinetic energy of unit as follows:

$$T\mathrm{e} = \frac{1}{2}\int_0^1 \mathrm{d}A\dot{r}^{\mathrm{T}}\dot{r}\mathrm{d}x = \frac{1}{2}\dot{e}^{\mathrm{T}} m_e \dot{e}$$

Where d for beam density; A cross-sectional area for beam; Above variables ". " said partial derivatives of time; Quality of m_e as a unit,

$$m_e = \int_0^l \mathrm{d}AS^{\mathrm{T}}S\mathrm{d}x = \mathrm{d}Al\int_0^l S^T S\mathrm{dY}$$

Set f is the force per unit volume array, referred to as the physical and virtual work for physical strength is:

$$\delta W = \int_0^l A\mathrm{W}r^{\mathrm{T}} f\,\mathrm{d}x = \delta e^{\mathrm{T}} Q_{\mathrm{e}}$$

The Q_e as a unit generalized force, $Q_{\mathrm{e}} = \int_0^l AS^{\mathrm{T}} f\mathrm{d}x$,

According to the geometric nonlinear theory, the axial strain is

$$\mathrm{X}_1 = \frac{1}{2}\left[\left[\frac{\partial r}{\partial x}\right]^{\mathrm{T}}\left[\frac{\partial r}{\partial x}\right] - 1\right] \qquad (2)$$

$S_1 = S'^{\mathrm{T}} S' = \frac{1}{l^2} S_{\mathrm{Y}}{}^{\mathrm{T}} S_{\mathrm{Y}}$, The shape function of S_{Y} is S partial derivative matrix of Y. For axial deformation of beam element is:

$$U_{\mathrm{e}1} = \frac{1}{2}\int_0^l EA\mathrm{X}^2 \mathrm{d}x$$

Then the stiffness matrix of and axial deformation can be written as

$$K_1 = \frac{1}{2} EAl\left[\int_0^l \left(e^{\mathrm{T}} S_1 e\right) S_1 \mathrm{dY} - \int_0^l S_1 \mathrm{dY}\right]$$

Bending deformation for the beam element is

$$U_{\mathrm{et}} = \frac{1}{2}\int_0^l EI\kappa^2 \mathrm{d}x$$

Among them: I is cleansing and moment of inertia; κ for the deformation of back rest axis curve curvature, due to the axial deformation of beam usually is less than bending deformation, is a small amount, so it is assumed that the beam element in axial deformation of the constant $\overline{f}$,so

$$\kappa = \left|\frac{\mathrm{d}^2 r}{\mathrm{d}s^2}\right| = \frac{1}{\overline{f}}\sqrt{e^{\mathrm{T}} S''^{\mathrm{T}} S'' e} \qquad (3)$$

On the type, the superscript "" "said x second order partial derivatives; the average deformation is

$$\overline{f} = \sqrt{\int_0^l f^2 \mathrm{dY}} = \sqrt{\int_0^l r'^{\mathrm{T}} r' \mathrm{dY}} = \sqrt{e^{\mathrm{T}} \overline{S}_{1e}}$$

So

$$U_{\mathrm{et}} = \frac{1}{2}\int_0^1 EI\kappa^2 \mathrm{d}x = \frac{1}{2} e^{\mathrm{T}} K_{\mathrm{t}} e$$

Among them, the stiffness matrix of K_{t} is related to the bending deformation,

$$K_{\mathrm{t}} = EIl\left[\frac{1}{\overline{f}^4}\int_0^l S''^{\mathrm{T}} S'' \mathrm{dY} - \frac{2}{\overline{f}^6} e^{\mathrm{T}}\left[\int_0^l S''^{\mathrm{T}} S'' \mathrm{dY}\right] e\overline{S}_1\right]$$

The overall stiffness matrix of the k_e unit of beam is $K_{ke} = K_1 + K_{\mathrm{t}}$ 。

A q for overall node coordinates array of single beam, n is beam unit number, B_{ke} for Boolean matrix of beam element, so $e = B_{ke} q$.

Among them, q for the 4 order array $(4n+4)\times 1$.Single beam, therefore, the overall quality of matrix, the elastic stiffness matrix and generalized force matrix, respectively are

$$m = \sum_{ke} B_{ke}^{\mathrm{T}} m_{ke} B_{ke}$$

$$K = \sum_{ke} B_{ke}^{\mathrm{T}} \left(K_1 + K_{\mathrm{t}}\right) B_{ke}$$

$$Q = \sum_{ke} B_{ke}^{\mathrm{T}} Q_{ke}$$

For N consisting of a beam system, upper right standard said beam label, for beam i dynamics equation is

$$m^{(i)} \ddot{q}^{(i)} + K^{(i)} q^{(i)} = Q^{(i)}$$

$$i = 1, \quad 2, \quad \cdots \quad N$$

2. Beam System Dynamics Equation

To form a beam generalized coordinates of nodes in the system, mass matrix, the elastic stiffness matrix and generalized force matrix，the dynamic equation of free beam system is $m\ddot{q}+Kq=Q$， Among them:

$$m=\begin{bmatrix} m^{(1)} & 0 & \cdots & 0 \\ 0 & m^{(2)} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & m^{(N)} \end{bmatrix}$$

$$K=\begin{bmatrix} k^{(1)} & 0 & \cdots & 0 \\ 0 & m^{(2)} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & m^{(N)} \end{bmatrix}$$

$$Q=\begin{bmatrix} Q^{(1)} \\ Q^{(2)} \\ \vdots \\ Q^{(N)} \end{bmatrix}, \qquad q=\begin{bmatrix} q^{(1)} \\ q^{(2)} \\ \vdots \\ q^{(N)} \end{bmatrix}$$

If with rotating hinge connections between the beam, and set up

$$q^{(i)}=\left[e_1^{(i)} \quad e_2^{(i)} \quad \cdots \quad e_{4n+1}^{(i)} \quad \cdots \quad e_{4n+4}^{(i)}\right]^{\mathrm{T}}$$

The kinematic constraint equations of the beam between is

$$e_{4n+1}^{(i)}=e_1^{(i+1)}, \qquad e_{4n+2}^{(i)}=e_2^{(i+1)}$$

$$i=1, \quad 2, \quad \cdots \quad N-1$$

$\left(e_{4n+1}^{(i)} \quad e_{4n+2}^{(i)}\right)$ for the i beam and the right endpoint of absolute coordinates; $\left(e_1^{(i+1)} \quad e_2^{(i+1)}\right)$ for the $i+1$ girder left endpoint of the absolute coordinates. Then, the number of independent elements for Q is

$$\bar{n}=(4n+4)N-2(N-1)=(4n+2)N+2$$

Assume that Q node coordinates W array for independence, can be expressed as

$$\bar{q}=\left[\cdots \quad e_1^{(i)} \quad e_2^{(i)} \quad \cdots \quad e_{4n-1}^{(i)} \quad e_{4n}^{(i)} \quad e_{4n+3}^{(i)} \quad e_{4n+4}^{(i)} \quad e_1^{(i+1)} \quad e_2^{(i+1)} \quad \cdots\right]$$

The relationship between q and $\bar{q}$ is $q=D\bar{q}$, Among them, D is the $(4n+1)N\times\bar{n}$ order sparse array. The dynamic equation of beam system is

$$\bar{m}\ddot{\bar{q}}+\bar{K}\bar{q}=\bar{Q}$$

$$\bar{m}=D^{\mathrm{T}}mD; \quad \bar{K}=D^{\mathrm{T}}KD; \quad \bar{Q}=D^{\mathrm{T}}Q.$$

3. Numerical Examples

As shown in figure 2, flat beam by horizontal position without falling velocity, A side for rotating A, B side free. The parameters of the beam is: d=2766.67kg/m3, l=1.8 m, A=25cm3, E=68.95 GPa, I=0.130cm^4, acceleration of gravity is 9.81 m/s2, the beam is divided into four units.

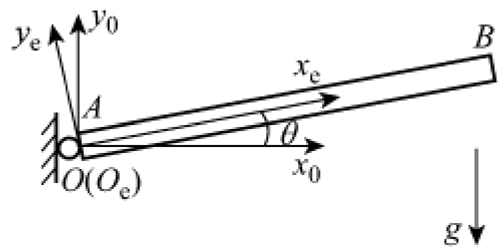

Fig 2: Single Pendulum

Left endpoint of the beam at A boundary condition for $e_1^{(1)} = 0, e_2^{(1)} = 0$ .In order to facilitate comparing with an approximate mixed coordinates method, the absolute displacement into the lateral deformation of the endpoint, Put θ as beam rotation angle of floating coordinate $O_e x_e y_e$ system, $e_3^{(1)}$、 $e_4^{(1)}$ slope for A point.

Then

$$\sin\theta = \frac{e_4^{(1)}}{\sqrt{\left(e_3^{(1)}\right)^2 + \left(e_4^{(1)}\right)^2}}$$

$$\cos\theta = \frac{e_3^{(1)}}{\sqrt{\left(e_3^{(1)}\right)^2 + \left(e_4^{(1)}\right)^2}}$$

Set A is $O_e x_e y_e$ coordinate system about the direction cosine matrix about the absolute coordinate system, type u_1 ， u_2 respectively, of the longitudinal and transverse deformation of beam end, then

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = A^{\mathrm{T}} \begin{bmatrix} r_1(l) \\ r_2(l) \end{bmatrix} - \begin{bmatrix} l \\ 0 \end{bmatrix}$$

Figure 3 for this article is absolutely node coordinates method to calculate the direction of the beam end y_0 absolute displacement. By figure 3 shows, compared with the absolute displacement due to deformation is small, take different modulus of elasticity, the absolute displacement basic same, explain the influence of the elastic deformation of absolute displacement is smaller. Figure 4 is absolutely node coordinate method and an approximate mixed coordinates method to calculate the lateral deformation of the beam of the endpoints. By figure can be seen, when $E = 6.895\text{GPa}$, due to the lateral deformation is small, the paper definitely node coordinate method and an approximate mixed coordinates method to calculate the lateral deformation of the beam of the endpoints; the maximum lateral deformation when $E = 6.895\text{GPa}$ and $E = 68.95\text{GPa}$ are 10 times, in line with the expected results, and the lateral deformation increase. An approximation of the hybrid coordinate method due to ignore the deformation in dynamic equation of higher order item, lateral deformation from the start because cannot satisfy the calculation accuracy and numerical divergence, absolute coordinates method due to the dynamics in this paper considering the deformation of high order term in the equation, the lateral deformation of beam endpoint convergence, shows the tendency of up and down shocks.

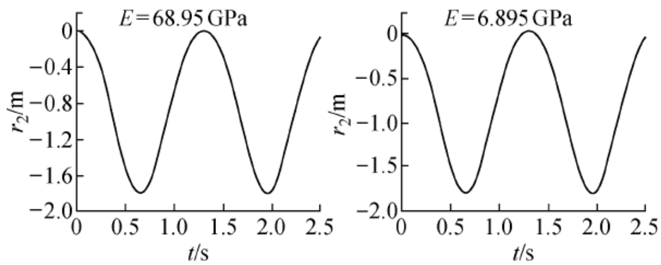

Fig 3: Absolute Displacement of the Beam tip in y_0 Direction

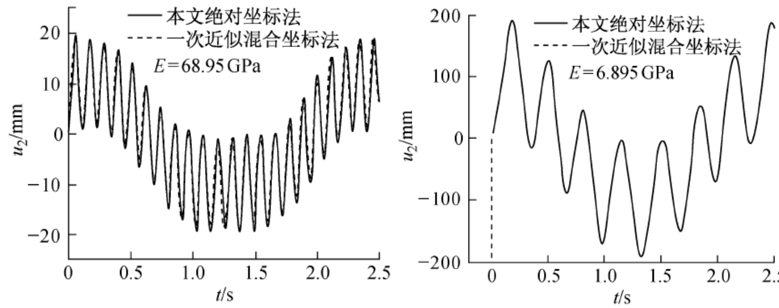

Fig 4: Transverse Deflection of the Beam Tip

In order to further verify the results, the energy analysis. The system provides a conservative system, in the process of swing beam, should accord with energy conservation, namely the total kinetic energy, elastic deformation energy, the conservation of the sum of the gravitational potential energy. In this example, the initial position, and take the overall coordinate system origin at point A so total energy conservation should be zero. That is

$$T + U + U_g = 0$$

In the type, T is kinetic energy; U_g is the gravitational potential energy; U is elastic deformation energy. Figure 5 for energy curve of absolute coordinates method by the chart shows, in this paper, absolute coordinates method to calculate the total energy constant is zero. Conform to the law of conservation of energy, the absolute node coordinates method is proved to be correct. When $E = 6.895\text{GPa}$ total energy remains zero, thus large deformation case absolute value of node coordinates method is accurate.

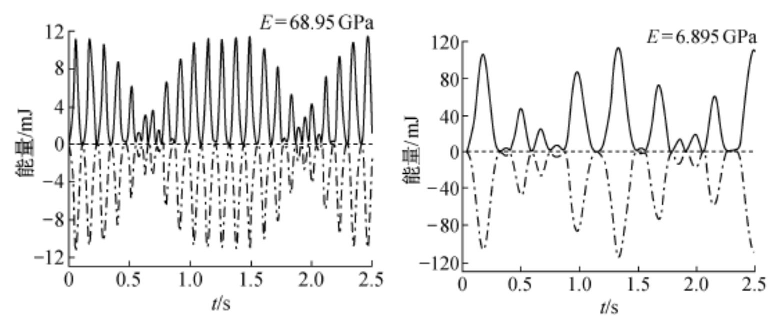

Fig 5: Energy of the Beam

In small deformation, with an approximate mixed coordinates method generalized coordinates will be cut by modal truncation method, computational efficiency is higher. In this example, an approximate mixed coordinates method for calculation of 10, absolute coordinates method of calculation time was 80, so in the case of small deformation should try to use a similar to the hybrid coordinate method. Example 2. By the horizontal position without falling velocity of the flexible double pendulum, two beams of physical parameters are the same as the case 1, $E = 68.95\text{GPa}$ .Figure 6 for point B the y_0 direction absolute displacement, figure 7 for the system energy curve.By the figure shows, sports system in the process of the total energy constant is zero, conform to the law of conservation of energy, can attest to the absolute node coordinates method in the flexible multi-body system dynamics problem has higher calculation accuracy.

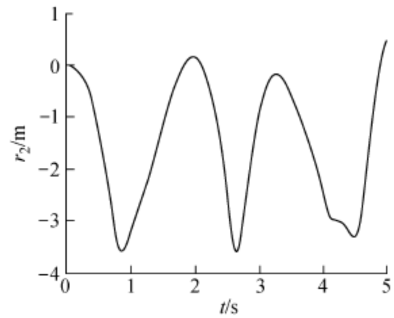

Fig 6: Absolute Displacement of Point B ing y_0 Direction

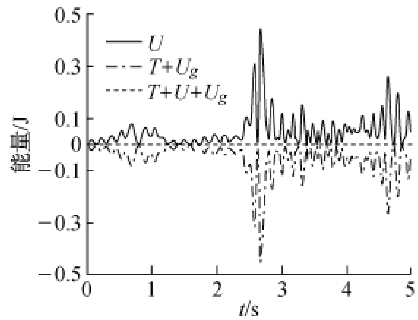

Fig 7: Energy of the Beam System

4. Conclusions

(1) A similar mixed coordinates method is only suitable for small deformation condition.

(2) Absolute node coordinates method is suitable for the small deformation condition, and is suitable for large deformation, and the result is quite accurate.

(3) In small deformation, with an approximate mixed coordinates method generalized coordinates will be cut by modal truncation method, computational efficiency is higher, so in the case of small deformation should try to use a similar to the hybrid coordinate method.

References

Kane T R，Ryan R R，Banerjee A K。Dynamics of a cantilever beam attached to a moving base [J].Journal of Guidance Control and Dynamics,1987,10(2):139-150.

刘锦阳，洪嘉振.大范围运动空间梁的刚柔耦合动力学模型[J].上海交通大学学报，2003, 37（4）：532-534

Shabana A A. Finite element incremental approach and exact rigid body inertia [J].Journal of Mechanical Design.1996,118:171-188.

Omar M A, Shabana A A.A two-dimensional shear deformable beam for large rotation and deformation problems[J].Journal of Sound and Vibration 2001,243(3):565-576.

Shabana A A, Hussien H A, Escalona J L. Application of the absolute nodal coordinate formulation to large rotation and large deformation problems [J].Transactions of the ASME, 1998, 120:188-195.

Berneri M, Shabana A A. Development of simple models for the elastic forces in the absolute nodal coordinate formulation [J].Journal of Sound and Vibration 2000, 235(4):539-565.

Yu L., Zhao Z. H., Ren G. X. Multibody Dynamic Model of Web Guiding System With Moving Web[J]. Journal of Dynamic Systems Measurement and Control-Transactions of the Asme, 2010,132(5).

Mohamed A. N. A., Shabana A. A. A nonlinear visco-elastic constitutive model for large rotation finite element formulations[J]. Multibody System Dynamics, 2011, 26(1):57-79.

Matikainen M. K., Valkeapaa A. I., Mikkola A. M., Schwab A. L. A study of moderately thick quadrilateral plate elements based on the absolute nodal coordinate formulation [J]. Multibody System Dynamics, 2014, 31(3):309-338.

Hu W., Tian Q., Hu H. Y. Dynamic simulation of liquid-filled flexible multibody systems via absolute nodal coordinate formulation and SPH method[J]. Nonlinear Dynamics, 2014, 75(4):653-671.