Geostatistic Applied to Spatial Modeling of Hypsometric Relationships in Forest Stands

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Abstract

Height trees can be estimated more realistically if the structure of spatial dependence is considered in hypsometric relationships. Thus, the aim of this study was to apply geostatistical techniques to modeling the spatial patterns and estimate total height in teak stands. Average values of total height (H) and diameter at 1.3 m above ground (DBH) were obtained from 273 sample units at the second and sixth years, after selective thinning. Four models were fit using traditional hypsometric relationships. Also, geostatistical analyses were used to model the spatial patterns of height, as well as cross-semivariograms to estimate the height when correlated with DBH. Traditional modeling was more accurate than simple geostatistical analysis; however, including the spatial variability of DBH, results were statistically superiors to map the height. Cross-semivariogram and cokriging analyses identified the spatial correlation of the height with DBH and described the spatial variability of young and post-thinning stands.

Keywords: Height/diameter ratio, Spatial variability, Geostatistical modeling. Kriging. Cokriging

1. Introduction

In order to reduce time and costs of forest inventories, hypsometric models are commonly used to estimate the height of individuals that cannot be measured directly, for the subsequent determination of the volume. However, the hypsometric relationship in forest stands is sensitive to several factors, such as age and silvicultural practices [6, 21, 25, 29].

In general, height growth of young trees is more pronounced and with high variability, which results in high errors for the estimates, while thinning tends to disfigure the correlation between the variables height and diameter 1.3 m above the ground, causing loss to efficiencies of modeling [5, 6, 21].

Besides these facts, traditional methods do not consider the spatial relation between the sampling units, which are affected, generally, by site characteristics and silvicultural practices in forest stands [24, 25].

Thus, as it increases the need for detailed information for the efficient forest managements, consolidates the use of geostatistical techniques [27], that are based on spatial functions of regionalized variables that allow to predict values at locations not sampled and mappings [7, 15, 18]. The final aim is more localized interventions, increased efficiency of cultural treatments and reduction of production costs [34].

Different the classical statistics, geostatistics is based on the premise that there is independence between the sampling points, and each features a probability distribution of occurrence of values that characterize the spatial dependence that statistically corresponds to the population which are extracted from representative samples [12, 38].

Geostatistics is based on the theory of regionalized variables [22], which defines the regionalized variable as a numerical spatial function of a structured space phenomenon, and the semivariance as its basic statistical measure, by which is measured the spatial structure between successive sampling points separated by distances [1, 9, 12].

Therefore, when considering the structure of spatial dependence in hypsometric relationships, adequate models are obtained to describe the total height, and to achieve more realistic estimates in unsampled locations [28]. Thus, the aim of this study was to apply geostatistical techniques to modeling the spatial patterns and estimate total height in teak stands.

2. Material and Methods

The study was conducted on 1,260 hectare of teak stands bounded by the geographical coordinates 16°09’00”S to 16°13’50”S Latitude and 56°21’00”W to 56°24’20”W Longitude. The climate is classified as Aw, according to the Köppen classification system, with average rainfall of 1,300 mm per year, average annual temperature of 25ºC. The topography is slightly undulated and the soil is classified as Haplic Eutrophic Planosol with a sandy-clay-loam texture.

A total of 273 permanent plots of 15 m x 30 m (450 m²) were placed in each stand according to the sampling intensity determined from the forest inventory and the coordinates for their geographic locations were recorded. Average values of total height (H) and diameter at 1.3 m above ground (DBH) were measured in the stand at two and six years old, after a selective thinning with 40% of trees per hectare removed. The descriptive statistics analysis of these variables is shown in Table 1.

Table 1: Descriptive Statistics of the Total Height (H) And Diameter at 1.3 M above Ground (DBH) After Two and Six Years Old of Teak Stands

Variable	Minimum	Mean	Maximum	Standard Deviation	Coefficient Variation	Kolmogorov-Smirnov test	Linear Correlation
2nd year							
H (m)	2.41	4.64	6.85	1.02	21.91%	0.049*	0.904
DBH (cm)	2.89	5.19	7.65	1.02	19.58%	0.058*	
6th year							
H (m)	11.80	13.88	15.91	0.85	6.15%	0.053*	0.728
DBH (cm)	13.36	16.85	20.08	1.37	8.11%	0.069*	

Where: * = normal distribution, at 5% significance level, by Kolmogorov-Smirnov test.

2.1 Traditional Modeling

Four models were fitted using traditional hypsometric relationships (Table 2) available in the forestry literature [3, 16, 20, 21] for the two age groups in the teak stands. The evaluation and selection criteria followed the highest adjusted coefficient of determination ($R^2_{aj.}$) and the lowest standard error of estimate ($S_{yx}\%$). The significance of the regression coefficients (β_i) and the graphical analysis of residuals plotted over function of diameter at 1.3 m above ground (DBH) were also evaluated.

When performing the inverse operation to obtain the variable of interest in the Stoffels and van Soest [32] and Curtis [11] logarithmic models, the logarithmic discrepancy in the estimate of the dependent variable was corrected by multiplying the estimated height by the Correction Factor (CF) as per the expression of Sprugel [31]:

$$CF=e^{0,5\,(S_{yx})^2} \quad (1)$$

Where: e = base of the natural log; and S_{yx} = standard error of the estimate.

2.2 Geostatistical Modeling

Geostatistical analysis was used to describe and model the spatial patterns of total height (H). The semivariogram is employed specifically as a mathematical tool that enables one to study the spatial dispersion of a variable as a function of the distance between sampling units [2] and is represented by the expression:

$$\gamma(h)=\frac{1}{2N(h)}\sum_{i=1}^{N(h)}[Z(x_i)-Z(x_1+h)]^2 \quad (2)$$

Where: $\gamma(h)$ = semivariance of the variable $Z(x_i)$; h = distance; and N(h) = number of points pairs measured of $Z(x_i)$ and $Z(x_i + h)$ separated by a distance h.

Semivariances were determined between the equidistant sampling points, with the regularization of the sampling grid by means of an angular tolerance of 5°. This process was repeated in four directions in the spatial plane: 0° (S-N); 45° (NE-SW); 90° (E-W); and 135° (NW-SE), of which the average semivariance matrix was obtained between distances equivalent, and the sampling units pairs were computed (Table 2).

Table 2: Example of Average Semivariances Matrix Calculated on a Space Plan

Distance	Semivariance	Sampling units pairs
d_1	$\gamma(d_1)$	n_1
d_2	$\gamma(d_2)$	n_2
d_3	$\gamma(d_3)$	n_3
d_4	$\gamma(d_4)$	n_4
d_n	$\gamma(d_n)$	n_n

Where: d_i = distance (m) between sampling units pairs; and $\gamma(d_i)$ = average semivariance to distance d_i.

Moreover, in situations where it was evident the spatial correlation between total height and diameter at 1.3 m above ground, the height estimates were determined considering both variables in coincident geographical locations, through cross-semivariogram (3):

$$\gamma_{1,2}(h)=\frac{1}{2N(h)}\sum_{i=1}^{N(h)}[Z_1(x_{1i}+h)-Z_1(x_{1i})]\,[Z_2(x_{2i}+h)-Z_2(x_{2i})] \quad (3)$$

Where: $\gamma_{1.2}(h)$ = semivariance of the variables Z_1 and Z_2; h = distance; and N(h) = number of points pairs measured of Z_1 and Z_2 separated by a distance h.

For the semivariance estimates in any distances between samples, were tested the spherical (4), exponential (5) and gaussian (6) geostatistical models with the aid of the computer program Geoest [35] and spreadsheet software.

$$\gamma(h)=C_0+C\left[\left(\frac{3}{2}\right)\left(\frac{h}{A}\right)-\left(\frac{1}{2}\right)\left(\frac{h}{A}\right)^3\right] \quad (4)$$

$$\gamma(h)=C_0+C\left(1-e^{-h/A}\right) \quad (5)$$

$$\gamma(h)=C_0+C\left(1-e^{-h^2/A^2}\right) \quad (6)$$

Where: $\gamma(h)$ = semivariance of the variable $Z(x_i)$; h = distance; C_0 = nugget effect; C = sill; and A = range.

Semivariogram structure was composed of nugget effect (C_0), which corresponds to the semivariance value at a distance zero and indicates the random variation of the data; sill (C), which is the stable value of the semivariogram approximately equal to the variance of the data; contribution (C_1), which is given by the difference between sill (C) and nugget effect (C_0); and range (A), which is defined as distance limit which the sampling units are correlated [36].

Weighted least squares method was used for the adjustments of the semivariogram. This method aims to minimize the weighted sum of squared deviations (WSSD), where the squared differences between the semivariances observed and semivariances estimated are weighted according to the number of pairs used for calculation of the average semivariances in each equidistant distance composing the semivariograma [4, 23].

Best fit selection was based on the minimum weighted sum of squared deviations (WSSD), the highest coefficient of determination (R^2), and the cross-validation that ideally provides a linear coefficient equal to zero, angular coefficient equal to one, and coefficient of determination of cross-validation (R^2_{cv}) equal to one. Furthermore, in order to verify the presence of anisotropy, the semivariograms were oriented at 0º relative to the X-axis, 90° from the Y-axis and at 45° and 135° on the diagonals [36]. In obtaining semivariances and distances that make up the semivariogram, neighborhood analysis was conducted on the 4, 8, 12, 16 and 20 neighbors. Finally, spatial dependence degree (DD) was calculated as per Cambardella et al. [8] and classified as: strong for DD ≤ 25%, moderate for 25% < DD ≤ 75%, and weak for DD > 75%.

Spatial interpolation was executed by ordinary point kriging (7), or by ordinary point cokriging (8) in fitting the cross-semivariograms, which considered the spatial dependence and estimate without bias and with minimum variance, and enabled thematic maps to be prepared [10]. These maps were generated with the aid of program Surfer 9.0 demo version [17] using the mean and standard deviation of total height for each assessment period to determine the classes.

$$Z^*_{KO}(x_0) = \sum_{i=1}^{n} \lambda_i \, [Z(x_i)] \qquad \text{Ordinary kriging} \qquad (7)$$

$$Z^*_1(x_0) = \sum_{i=1}^{n_1} \lambda_{1i} \, Z_1(x_{1i}) + \sum_{i=1}^{n_2} \lambda_{2i} \, Z_2(x_{2i}) \qquad \text{Ordinary cokriging} \qquad (8)$$

Where: Z^*_{KO} = estimator; λ_i = weights; $z(x_i)$ = observed value; $Z^*_1(x_0)$ = estimated primary variable in point x_0; Z_1 e Z_2 = primary and secondary variables, respectively; and n = neighbors.

The technique of Lagrange Multipliers (9) was used to determine the values of the weights (λ_i) in the estimates of non-sampled locations [36, 37], since each sampling unit contributes in percentage distinct in the estimates of the points not sampled. Finally, the process involved inversion matrix A and multiplication by matrix B to determine λ, respecting the condition: $\sum \lambda_i = 1$. Thereafter, the process was repeated in all places to estimate the total height.

$$\underbrace{\begin{bmatrix} \gamma(x_1,x_1) & \gamma(x_1,x_2) & \cdots & \gamma(x_1,x_n) & 1 \\ \gamma(x_2,x_1) & \gamma(x_2,x_2) & \cdots & \gamma(x_2,x_n) & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \gamma(x_n,x_1) & \gamma(x_n,x_2) & \cdots & \gamma(x_n,x_n) & 1 \\ 1 & 1 & \cdots & 1 & 0 \end{bmatrix}}_{[A]} \times \underbrace{\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \\ \mu(x_0) \end{bmatrix}}_{[\lambda]} = \underbrace{\begin{bmatrix} \gamma(x_1,x_0) \\ \gamma(x_2,x_0) \\ \vdots \\ \gamma(x_n,x_0) \\ 1 \end{bmatrix}}_{[B]} \qquad (9)$$

Where: $\gamma(x_N, x_N)$ = semivariances estimated between observed sample points; λ_n = weights; $\mu(x_0)$ = estimated value in the not sampled point (x_0); and $\gamma(x_n, x_0)$ = semivariances estimated between sampling points and not sampled locations (x_0).

4. Results and Discussion

In general, adjustment obtained using traditional models of hypsometric relationships were statistically similar (Table 3) for both age groups of teak stands, with significant regression coefficients (β_i), at the 5% probability level, except for the Trorey model in the six year old. Adjusted coefficient of determination ($R^2_{aj.}$) resulted values around 0.81 in the second year and 0.52 in the sixth year, whereas the standard error of estimate (S_{yx}%) resulted values around 9% and 4%, respectively at the second and sixth years, as showed in Table 3.

Table 3: Statistical Parameters of the Hypsometric Relationship Traditional Models

No.	Denomination	Model	β_0	β_1	β_2	$R^2_{adj.}$	S_{yx}%
2^{nd} year							
1	Trorey [33]	$H = \beta_0 + \beta_1 DAP + \beta_2 DAP^2$	-0.9763*	1.2833*	-0.0374*	0.818	9.34
2	Henricksen [19]	$H = \beta_0 + \beta_1 \ln(DAP)$	-2.4847*	4.3807*		0.813	9.47
3	Stoffels and van Soest [32]	$\ln(H) = \beta_0 + \beta_1 \ln(DAP)$	-0.1733*	1.0340*		0.817	9.40
4	Curtis [11]	$\ln(H) = \beta_0 + \beta_1 (1/DAP)$	2.4633*	-4.7465*		0.813	9.47
6^{th} year							
5	Trorey [33]	$H = \beta_0 + \beta_1 DAP + \beta_2 DAP^2$	7.9762 ^{ns}	0.2421*	0.0064*	0.527	4.23
6	Henricksen [19]	$H = \beta_0 + \beta_1 \ln(DAP)$	-7.2019*	7.4744*		0.525	4.24
7	Stoffels and van Soest [32]	$\ln(H) = \beta_0 + \beta_1 \ln(DAP)$	1.0834*	0.5478*		0.529	4.23
8	Curtis [11]	$\ln(H) = \beta_0 + \beta_1 (1/DAP)$	3.1613*	-8.9120*		0.523	4.25

Where: ^{NS} = not significant; and * = significant at 5%.

Higher values for the standard error of estimate (S_{yx}%), at the second year (Table 3), was the result of the high variability in height growth of young forest stands, as observed by Bartoszeck et al. [6] for bracatinga and by Donadoni et al. [13] for tropical pines, while lower coefficients of determination ($R^2_{aj.}$), at the sixth year (Table 2), showed a reduction in the linear correlation between total height and DBH after thinning, as observed by Barros et al. [5] in *Pinus oocarpa* Schiede, who stated that thinning promotes a change in forest structure and height homogenization, such that many trees with different diameters have similar total height, and, in this case, the estimated values tend their arithmetic average, with reduced standard error of the estimate (S_{yx}%).

Total height residuals, obtained by Trorey model in the second year (Figure 1A) and Stoffels and van Soest model in the sixth year (Figure 1C), reveals homogeneous residuals distribution. In the hypsometric curve, estimated in the second year (Figure 1B), was evidenced the ascending behavior of the height/diameter curve, as well as the slope and concavity characteristic of young forest stands. Meanwhile, the curve flattening and its change to larger diameter classes, in the sixth year (Figure 1B), corroborated the dynamic effect of hypsometric relationship over time [3, 6, 16].

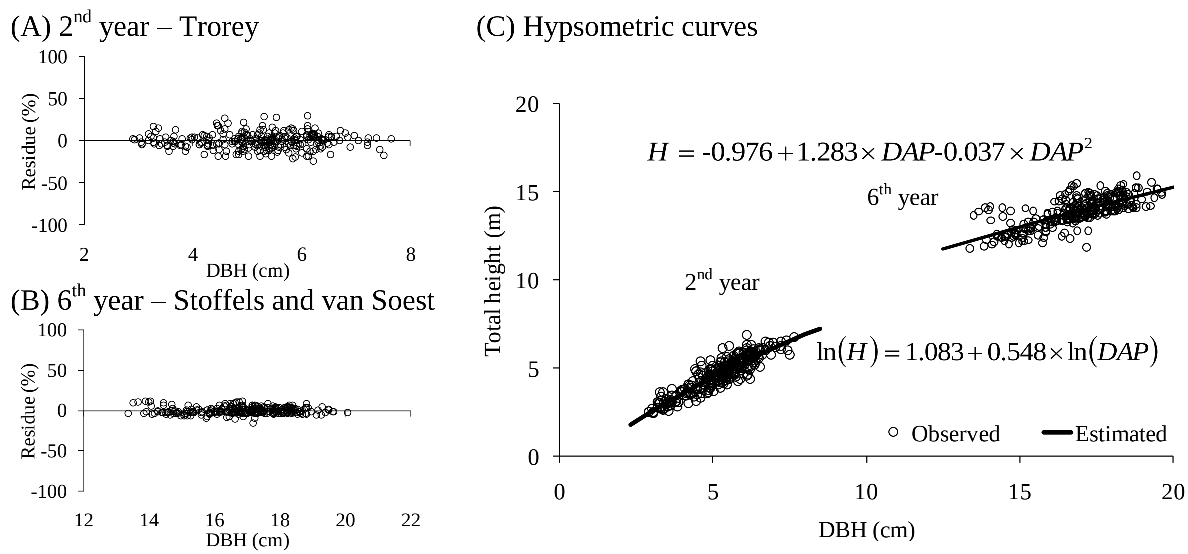

Figure 1: Residuals Distribution (A and B) and Hypsometric Curves (C) Estimated by Hypsometric Traditional Models

Total height (H) and diameter at 1.3 m above the ground (DBH) semivariograms, and the cross-semivariogram (H x DBH), were spatially dependents in the geostatistical modeling (Table 4).

Table 4 - Semivariograms Parameters for Total Height (H), Diameter at 1.3 M Above Ground (DBH) and the Relationship between Total Height and Diameter At 1.3 M Above Ground (H X DBH)

No.	Variable	Model	C_0	C_1	A (m)	DD (%)	R^2	WSSD
2nd year								
1		Spherical	0.399	0.644	2,035	38.2	0.986	0.0005
2	H	Exponential	0.249	0.831	2,312	23.1	0.984	0.0006
3		Gaussian	0.510	0.538	1,792	48.7	0.980	0.0006
4		Spherical	0.431	0.534	1,401	44.7	0.946	0.0012
5	DBH	Exponential	0.473	0.538	2,312	46.8	0.955	0.0018
6		Gaussian	0.518	0.448	1,198	53.6	0.946	0.0012
7		Spherical	0.401	0.641	2,035	38.5	0.986	0.0005
8	H x DBH	Exponential	0.261	0.813	2,312	24.3	0.979	0.0007
9		Gaussian	0.512	0.534	1,792	48.9	0.980	0.0005
6th year								
10		Spherical	0.313	0.387	1,421	44.8	0.872	0.0016
11	H	Exponential	0.359	0.369	2,312	49.3	0.787	0.0024
12		Gaussian	0.378	0.323	1,208	53.9	0.861	0.0017
13		Spherical	0.833	0.682	1,326	23.1	0.822	0.0045
14	DBH	Exponential	0.769	0.793	1,838	49.2	0.911	0.0034
15		Gaussian	0.941	0.575	1,132	62.1	0.892	0.0044
16		Spherical	0.410	0.323	1,243	23.1	0.822	0.0026
17	H x DBH	Exponential	0.089	0.642	837	12.1	0.771	0.0025
18		Gaussian	0.433	0.297	932	59.4	0.737	0.0027

Nugget effect (C_0) is the unexplained variance, which is caused by errors or variations that cannot be identified [36]. Thus, low values (less than one) were observed for C_0, indicating satisfactory fit of the semivariograms (Table 4), while the values of range (A), between 1,198 m and 2,312 m in the second year and between 837 and 2,312 m in the sixth year (Table 4), suggest high spatial heterogeneity. In general, fits were obtained with moderate spatial dependence degrees (DD), which implies in a specific spatial analysis of these dendrometric variables [27].

Coefficients of determination (R^2), obtained at second year, were greater than 0.94 and greater than those observed at sixth year ($0.737 \leq R^2 \leq 0.911$), while, in this latter age group, the weighted sum of squared deviations (WSSD) were greater, ranging from 0.0016 to 0.0045, compared to the range of 0.0005 to 0.0018 obtained at the second year (Table 4). This indicates that changes in spatial structure of forest stands, by thinning, tend to alter the spatial continuity of their dendrometric characteristics.

For cross-validation (Table 5), the predominance of spherical model was observed at the second year, as well as for the total height (H) variable at the sixth year. On the other hand, the exponential model and the use of four and eight neighbors were most representative for the other cases.

Table 5: Cross-Validation Parameters of Geostatistical Fits Selected for Total Height (H), Diameter at 1.3 M Above Ground (DBH) and the Relationship between Total Height and Diameter At 1.3 M Above Ground (H X DBH)

No.	Variable	Selected Model	Neighbors	Coefficient		R^2_{vc}	S_{yx}%
				Linear	Angular		
2nd year							
1	H	Spherical	8	2.226	0.522	0.537	14.94
2	DBH	Spherical	4	2.476	0.526	0.493	13.96
3	H x DBH	Spherical	4 (H) and 8 (DBH)	0.087	0.979	0.908	6.87
6th year							
4	H	Spherical	8	7.458	0.464	0.480	4.45
5	DBH	Exponential	20	9.438	0.441	0.453	5.97
6	H x DBH	Exponential	4 (H) and 4 (DBH)	0.982	0.929	0.630	4.41

The selected fits resulted in linear coefficients between 0.087 and 9.438; angular coefficients between 0.441 and 0.979; coefficients of determination of the cross-validation (R^2_{vc}) between 0.453 and 0.908; and standard error of estimate (S_{yx}%) between 4.41 and 14.94%. Thus, appropriate fits from semivariograms were obtained for estimates of the total height in unsampled locations, especially, when using cross-semivariogram in the H x DBH relationship, which resulted in the cross-validation parameters closer to the theoretical ideal values.

Thus, by plotting the residues (%) of the estimated total height (Figure 3), the largest residual dispersion was observed for the simple geostatistical modeling (Figures 2B and 2E). While, with the geostatistical analysis of H x DBH, lower residue variability was observed (Figures 2C and 2F), which indicate that the spatial correlation is a characteristic of these dendrometric variables [24, 25, 28], and, thus, better performance is achieved, compared to traditional modeling (Figures 2A and 2D).

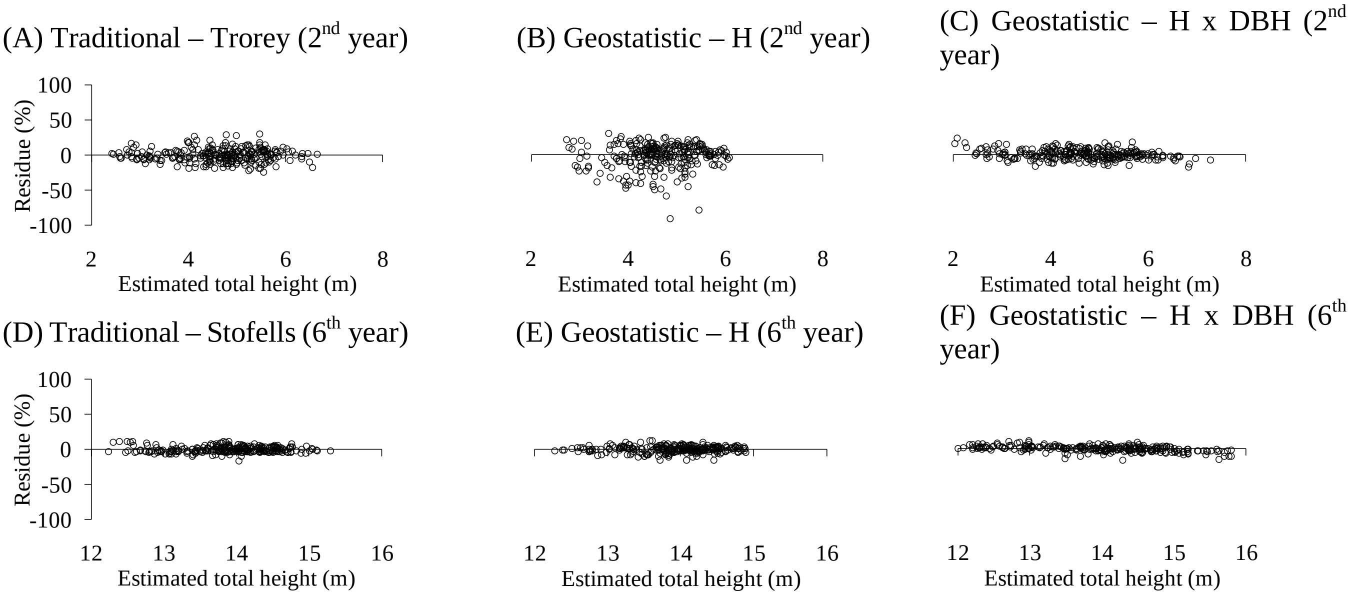

Figure 2: Residuals Distribution of Total Height Estimated by Traditional and Geostatistical Methods

With the semivariogram H x DBH was observed reduced scattering of values around the midline, showing the appropriate characteristics that resulted in satisfactory adjustments for spatial estimates of the total height (Figures 3A and 3C). Also, with anisotropic analysis was possible to identify the absence of significant structural differences of directional semivariograms (Figures 3B and 3D), admitting, thereby, the existence of isotropic behavior in selected fits.

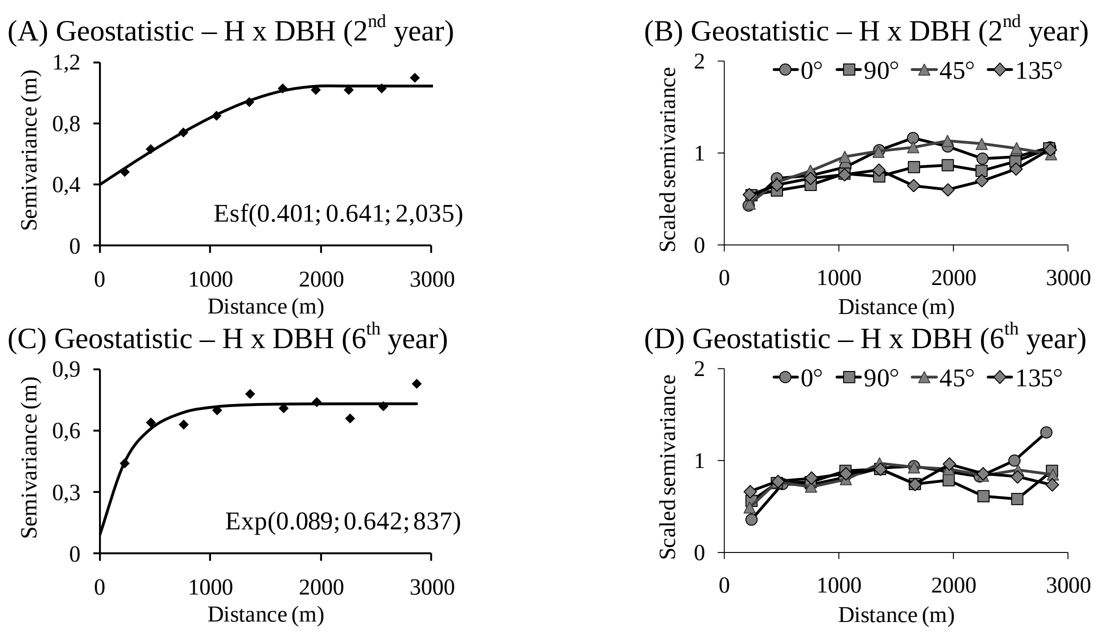

Figure 3: Theoretical and Directional Semivariograms for the H X DBH Geostatistical Relationship

Thus, after fits selected and spatial dependence and isotropy phenomenon observed (Figure 3), the ordinary cokriging interpolation was proceeded for mapping the total height, spatially correlated with the diameter of 1.3 m above the ground, at second (Figure 4A) and sixth years (Figure 4B).

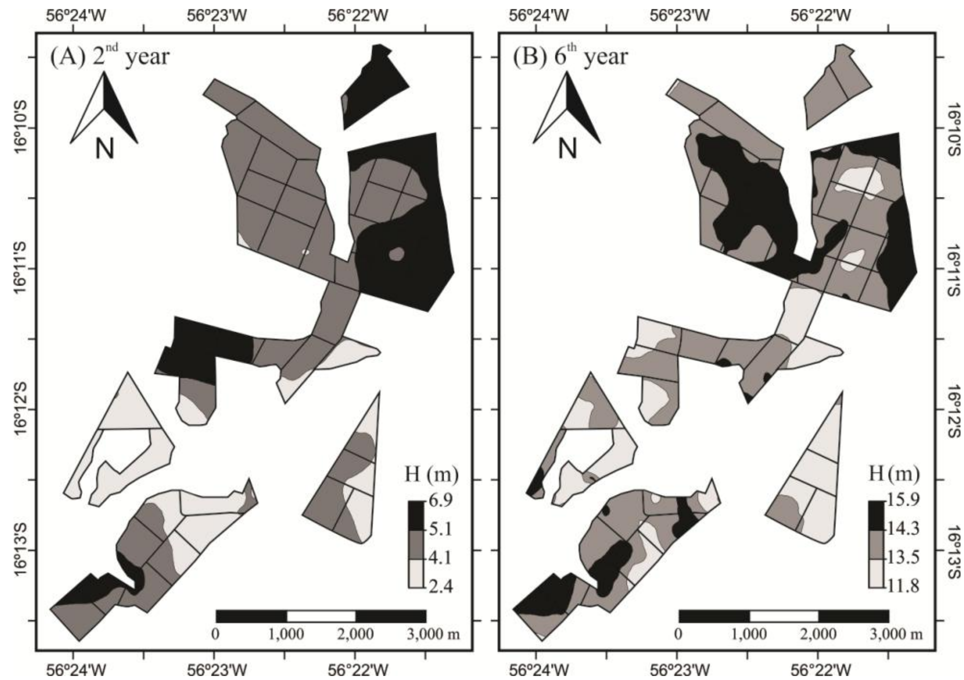

Figure 4: Thematic Maps of the Spatial Total Height Distributions in Teak Stands, At Two (A) And Six (B) Years Old

High heterogeneity of the total height was observed in thematic maps (Figures 4A and 4B), noting the distortion of the spatial continuity of this variable after reduction density of stands with the application of selective thinning in the fifth year of age. Furthermore, the aging of plantations and the site variability are factors that tend to affect the hypsometric relationship in forest stands [5, 6, 21].

These factors are responsible for deteriorating the linear correlation of the total height with a diameter of 1.3 m above the ground and, thus, harm the hypsometric relationship modeling to forest stands [5, 30].

Added to this, the data series stratification by height/diameter groups or by diameter classes are sometimes ineffective.

Moreover, with the mappings (Figures 4A and 4B) was attested the effectiveness of spatial coestimation the total height by diameter to 1.3 m above the ground, and it is also possible to apply when the main variable is the lower sampling density secondary variable. Therefore, if the sampling intensity of a forest inventory does not satisfy the requirements for geostatistical modeling of the total height, temporary units such as Bitterlich and Prodan points should be installed as well as methods commensurate with the size or distance, are appropriate for quickly and cost-effectively obtaining estimates in the auxiliary variable diameter at 1.3 m above ground.

5. Conclusions

Traditional modeling of hypsometric relationships is more accurate than simple geostatistical analysis of the total height, however, including spatial dependence of the diameter at 1.3 m above ground in the geostatistical modeling, results are statistically superiors to map the total height in forest stands.

Cross-semivariogram and ordinary cokriging methods identify the spatial correlation between total heights with diameters and, therefore, describe the spatial variability of the height in young and post-thinning stands. Such techniques allow the estimation of the dendrometric variable as a function of another more easily obtainable.

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