The Fallacy Regarding Newton's First Law: Its Non Axiomatic Nature

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Abstract

This paper investigates if Newton's first law of inertia fulfills the requirements in order to be considered as an axiom of Newtonian mechanics as presented in the Principia. The fact that it does not fulfill its alleged purpose of providing an error-free way of empirically identifying inertial reference frames, its inability to provide additional information apart from that which is already provided by the second law alone and, the fact that it may be deduced from the second law logically disqualifies it as an axiom of Newtonian mechanics.

Keywords: Newton's laws, axiom, theory, law, definition, inertia, inertial reference frame, fictitious force

1. Newton's First Law as an Axiom

Since the presentation of the *Principia* in 1687, Newton's first law has been accepted as one of the fundamental laws or axioms in the theory of Newtonian mechanics. This idea has rooted so deeply that even today's textbooks on physics present the law as one of these fundamental principles or axioms. However, when the nature of 'axiom' as a logical independent statement[1] is examined, it is doubtful that this law may be considered such because it provides no additional information to the theory than the one provided by the second law alone as we shall see in this paper.

It is typical to find in general physics textbooks statements like this one:

"Reference frames where the law of inertia does not hold … are called non inertial reference frames. How can we be sure a reference frame is inertial or not? By checking to see if Newton's first law holds. Thus Newton's first law serves as the definition of inertial reference frames" (Giancoli).

The first law is associated with the concept of inertia and with the identification of inertial reference frames in the context of Newton's classical mechanics in many discussions in which it is presented. It is argued that the empirical confirmation of this law in a certain reference frame, that is, the confirmation of the fact that an object maintains a constant velocity when no forces act on it, is a warranty that the frame is an inertial one. However, I claim that this empirical confirmation is no warranty that the frame considered is an inertial one in the context of Newtonian mechanics unless it is a fact that all the forces acting on the system have been identified and included in the analysis. However, as I shall show in section 3 of this essay, experience does not assure this fact.

2. The Role of Fictitious Forces in the Identification of Inertial Reference Frames

The idea that fictitious forces arise in accelerated reference frames has been recognized as the criteria for categorically identifying the degree of inertial motion of reference frames in the context of Newtonian mechanics. However, more than 300 years after the publication of the *Principia* there is still some confusion regarding the conditions under which these fictitious forces will arise. These ideas regarding the generation of fictitious forces in accelerated frames have been discussed deeply. Let's clarify how this kind of forces show up in accelerated frames suggesting that the frame is non inertial in the Newtonian mechanics sense.

[1] In a closed logical system the logical independence of the fundamental principles is what makes them axioms in that logical system. In other words the fact that an axiom may not be deduced from another axiom is what gives them this status in the system.

Let's consider the motion of an object that moves under the influence of the force of gravity with respect to a certain frame of reference that is fixed to the Earth. If a ball is released from a certain height, it describes a vertical trajectory with respect to this frame. If the same object is then released from the same height a second time but from a car that is moving horizontally with constant velocity with respect to the Earth, then the observed trajectory in the car's reference frame looks the same as the vertical fall observed from the reference frame fixed to the Earth when the ball was released the first time. This time, in the Earth's reference frame, the observed trajectory is a parabola. Since the second law is confirmed no matter which frame is considered, and since there is no criteria to determine which of the two frames is at rest with respect to absolute space, if any, then it should be concluded that both are Newtonian mechanics inertial reference frames. These frames are also characterized as equivalent frames.

Now suppose that the ball is released from a car that is moving horizontally with a constant acceleration with respect to the Earth. In the frame of the Earth the observed trajectory is the same parabola already considered, but in the frame of the accelerated car, apart from the vertical accelerated free fall motion, which is due to the action of the force of gravity, there is an observed acceleration in the horizontal direction pointing backwards. To explain this, Newton's second law is invoked. Since the only identified force is the gravitational vertical pull downwards, the backwards horizontal acceleration forces the introduction of a fictitious force acting in this direction on the falling object so that the vector addition of the two forces, or net force acting on the ball, may be invoked to explain the observed curved path. This fictitious force leads to the introduction of the concept of non inertial reference frames as ones in which Newton's second law doesn't hold. On these grounds we identify the accelerated frame of the car as a non inertial one. In the Newtonian mechanics context a non inertial reference frame is a frame that is accelerating with respect to absolute space. In summary, it is alleged that in the context of Newtonian mechanics non inertial reference frames are identified because, in them, fictitious forces[2] arise. It is interesting to observe that every Newtonian mechanics non inertial reference frame has an acceleration that is not zero with respect to absolute space being this acceleration the same when calculated with respect to any inertial reference frame. That is, the accelerations are absolute. This is the reason why it can be said that velocities are relative to the inertial frame selected for the analysis but accelerations are absolute with respect to them.

3. The Argument

This argument shows that in the context of Newtonian mechanics the empirical confirmation of Newton's law of inertia is no warranty that the frame is a Newtonian Mechanics inertial frame. Let us give thought to the following *Gedanken* experiment. Suppose that a group of observers are inside a laboratory room big enough to carry out experiments for the purpose of testing Newton's first and second laws and have no way of looking out. For example, suppose that they are in a room that has no windows so that there is no way of identifying anything outside, for example, a massive body in case there is one. Let's define this room and all the objects inside it as our system. Now suppose that the room is moving with a constant velocity as illustrated in figure 1, for example, let's say it is moving through space in the absence of all external forces. In this case it is not difficult to show that inside the room everyone will agree on the fact that Newton's first law is confirmed. If no forces are exerted on the body being experimented with, it will move with a constant velocity. So the conclusion in this case, since Newton's first law is confirmed, is that the frame of the room is a Newtonian mechanics inertial frame.

Now suppose, as shown in figure 2, that the room is subject to a constant[3] gravitational force that sets the system in a state of free fall. In this case the observers inside the room are unaware of the presence of this gravitational force acting on the whole system so this force is not included in the analysis as one of the forces acting on the object being experimented with. They are also unaware of their acceleration so inside the room the empirical confirmation of Newton's first law will yield as a false result that their reference frame is a Newtonian mechanics inertial frame.

Up to this point, taking into account the results of this *Gedanken* experiment, it has been shown that the empirical confirmation of Newton's first law in a certain reference frame does not necessarily imply that the frame considered is a Newtonian mechanics inertial frame.

[2] It is in fact fictitious effects and not fictitious forces that are observed.

[3] In this *Gedanken* experiment the gravitational force F_g is assumed constant for sake of simplicity, but it can be easily seen that the argument is valid even considering a variable gravitational force that produces a variable free fall acceleration.

In order to assure the fact that the frame is a Newtonian mechanics inertial frame from the empirical confirmation of the law, it is a requisite that all the forces acting on the system must be identified, specially gravitational ones since, as has been shown, gravitational forces may produce free fall states in which fictitious effects do not show as they are supposed to do in accelerated frames of reference. The reason for this lies in the fact that a state of constant velocity under the influence of no forces is empirically undistinguishable from a state of free fall due to a gravitational force. For example, the only way of identifying the frame of the Earth as a non inertial Newtonian mechanics frame due to its motion around the Sun is not to observe arising fictitious effects in the frame of the Earth, but to observe that in fact there is a body that has a certain mass, the Sun, and then allude to the theory that states that there should be a gravitational force acting on the Earth. In this system the Earth suffers the greatest acceleration because it has a mass that is not significant when compared to that of the Sun.

Since the empirical confirmation of Newton's first law does not necessarily determine the degree of inertial motion of reference frames in the context of Newtonian mechanics as has been shown, it should be investigated if there is a way to determine so. We should consider now if the empirical test of Newton's second law may serve as a way of identifying Newtonian mechanics inertial reference frames from non inertial ones. This law may be tested by applying forces $\vec{F}_n$ to an object and finding out if the observed accelerations $\vec{a}_o$ are proportional to these applied forces. Now suppose that the whole system is again set in a state of constant velocity as shown in figure 3. In this case it can be easily seen that Newton's second law is confirmed empirically in the frame of reference of the room yielding as a correct conclusion that the frame selected is a Newtonian mechanics inertial frame.

For the last part of the *Gedanken* experiment the whole system is subject again to a state of free fall due to a gravitational force and the observers look for the confirmation of Newton's second law as illustrated in figure 4. Since the gravitational force and the acceleration it produces have not been identified, as in the case of the confirmation of the first law when the system was on a state of free fall (figure 2), the observers inside the room will yield again confirmatory results for the second law giving as a false conclusion that the frame of reference of the room is a Newtonian mechanics inertial frame. The failure to identify this frame as a non inertial one comes from the inability to recognize the external gravitational force and the state of free fall to which the whole system is subject to. The problem behind the identification of forces is that they are unobservable quantities. Experience does not assure their identification.

4. Fictitious Effects in Frames that Move with Free Falling Objects

Based on the preceding analysis it follows that fictitious effects that need the introduction of fictitious forces in order to be explained do not show in frames of reference moving with objects that are in a state of free fall. This is also true for reference frames that move with constant velocity with respect to these accelerated frames. If we on planet Earth were unable to identify the force of gravity that the Sun exerts on us, for whatever reason, no fictitious effects will show in the frame of reference of the Earth, not because they are unnoticeable, but because they are not generated in theory.

In the video Frames of Reference (Academic Film Archive of North America) there is a discussion about the fact that the Earth's reference frame is a Newtonian mechanics inertial frame but only approximately. This conclusion is drawn from the fact that there is indeed a fictitious force that is about .3% the force of gravity and in the opposite direction that generates from the fact that the Earth is rotating. However this force is not significant in relation to the force of gravity so the frame is for all practical purposes an inertial one. In the video there is a statement that clashes with the result obtained in the *Gedanken* experiment presented in the previous section. It is declared that "the acceleration of the Earth in its orbit is even smaller still and produces even smaller effects" (Academic Film Archive of North America) than those produced in the frame of the rotating Earth. But I have shown that this is not the case since, with respect to the Sun, the Earth is in a state of free fall and hence no fictitious effects should be generated in the frame of the Earth. Almost 300 years after the presentation of the *Principia* this misconception is still sustained.

5. Principia's Book I Scholium

Newton was aware of the difficulty in identifying inertial reference frames and he showed his concern in the *Scholium* presented in the *Definitiones* section at the beginning of the *Principia*.

He stated: *Motus quidem veros corporum singulorum cognoscere, & ab apparentibus actu discriminare, difficillimum est; propterea quod partes spatij illius immobilis in quo corpora vere moventur, non incurrunt in sensus. Causa tamen non est prorsus desperata. Nam suppetunt argumenta partim ex motibus apparentibus, qui sunt motuum verorum differentiæ, partim ex viribus quæ sunt motuum verorum causæ & effectus*[4](*Newtono*). Here it is relevant to clarify what Newton understood by 'true motion' and by 'apparent' or 'relative motion'. "*Rursus motus verus a viribus in corpus motum impressis semper mutatur, at motus relativus ab his viribus non mutatur necessario*"[5](*Newtono*).

It should be pointed here that these definitions, from which the Newtonian definitions of inertial and non inertial frames may be inferred, somewhat contrasts with Giancoli's quote in the sense that Newton proposes the test of the second law instead of the first. That is, he alludes to the absence or presence of a change in the state of motion due to the presence or absence of forces.

As can be seen from the first quote Newton categorically stated that, in order to distinguish apparent motions from true motions, it is a fact that we need the data related to the motion with respect to a certain reference frame but also the information regarding the forces that act on the system. It is important to keep in mind that the concept of absolute space is pragmatically useless or elusive, as Newton describes it, because empirically it may not be identified and therefore it is impossible to assure absolute motion with respect to it. However, it may be useful in clarifying theoretically the concept of inertial reference frame and how it is related to it.

Only in the case of rotational motion may absolute motion be recognized, but this does not clash against the idea that the confirmation of Newton's first law does not represent an error-free way for the identification of Newtonian mechanics reference frames:

"*Effectus quibus motus absoluti et relativi distinguuntur ab invicem, sunt vires recedendi ab axe motus circularis. Nam in motu circulari nude relativo hæ vires nullæ sunt, in vero autem et absoluto majores vel minores pro quantitate motus"*[6]*(Newtono).*

This idea is consistent with Mach's conclusion that only rotational motion of a body may be absolutely identified as non inertial since fictitious centrifugal effects are always generated in this case. For example, the motion of a Foucault pendulum placed at the Earth's North Pole would not swing in the same plane if the motion is referred to the frame of the Earth but it would remain in the same plane of swing relative to the frame of the stars. According to Mach "When, accordingly, we say that a body preserves unchanged its direction and velocity in space, our assertion is nothing more or less than an abbreviated reference to the entire universe" (Mach).

6. The Logical Structure of Theories

We have seen from the previous considerations that the first law does not provide empirically any additional knowledge or information to the analysis from that provided by the second law. Now I shall consider if from the point of view of logics the first law's introduction as one of the axioms is justified. In the twentieth century logical positivists of the Vienna Circle established the fundamentals of the modern philosophy of science. The common view of a scientific theory, according to them, is that of a closed logical system in which there is a relation of deducibility among the sentences that belong to the system. Many twentieth century philosophers, including non positivists, accepted and promoted this view. In every logical system there is a set of fundamental sentences that are accepted initially and serve as premises for the deduction of the other sentences or components of the logical system. These fundamental sentences, in the case of scientific empirical theories, are law-like statements that represent laws of nature.

[4] "It is indeed a matter of great difficulty to discover, and effectually to distinguish, the true motions of particular bodies from the apparent; because the parts of that immovable space, in which those motions are performed, do by no means come under the observation of our senses. Yet the thing is not altogether desperate; for we have some arguments to guide us, partly from the apparent motions, which are the differences of the true motions; partly from the forces, which are the causes and effects of the true motions" (Newton).

[5] "True motion suffers always some change from any force impressed upon the moving body; but relative motion does not necessarily undergo any change by such forces" (Newton).

[6] "The effects which distinguish absolute from relative motion are, the forces of receding from the axis of circular motion. For there are no such forces in a circular motion purely relative, but in a true and absolute circular motion, they are greater or less, according to the quantity of motion" (Newton).

In order for these laws to qualify as axioms in a certain theory they should fulfill a set of requirements among which there is a special property called ‘logical independence’ which is fundamental.

Logical independence of a certain axiom is simply the impossibility of deducing it from other statements that belong to the same logical system. This logical independence is recognized by the fact that the axiom being considered provides information to the logical system which otherwise would not be available if the axiom is left out of the logical system.

Another relevant aspect to be discussed regarding the logical structure of theories is the nature of a particular type of statement that is part of every logical system: the definition. Definitions are necessary in order to clarify concepts that are used within the logical system. For example, within the atomic theory the concept of ‘atom’ must be defined so that certain propositions related to it may be introduced with sense in the context of the theory. Within the theory of plate tectonics the concept of ‘plate’ must be introduced if the theory is to present propositions about the way tectonic plates interact. At this point it is important to mention that the conceptual difference between laws and definitions lies in the fact that laws are subject to being true or false. Definitions are not subject to being true or false because they are accepted by common agreement within a community. There is no sense in questioning if they are true or false.

Bringing Newton’s first law back to the discussion, the fact that it is only valid in a certain kind of frame of reference, that is, in inertial frames of reference, does not provide the definition of what an inertial frame of reference is. If an object on which no force is acting is observed to have a non zero acceleration then what proceeds logically is not to conclude that the frame is not a Newtonian mechanics inertial frame but to conclude that the law does not confirm. Only if within the context of the theory the concept of ‘inertial reference frame’ is defined can we say we have enough information in order to make the deduction that a certain frame is inertial or not depending on the result of the test of the law. Any definition belonging to a logical system must be presented categorically within it by introducing it as one of the premises of the system.

Let us analyze the definition that can be inferred from Giancoli’s quote. He suggests testing Newton’s first law so let’s propose the following definition: An inertial frame is one in which Newton’s first law holds. Under these conditions the deductive argument is as follows:

Premises:

1) A body persists in a state of constant velocity unless there is a force that alters this motion (Newton’s first law).
2) An inertial frame is one in which the first law holds (definition proposed).
3) Newton’s first law holds in the frame selected (empirical data).

Conclusion:

The selected frame is inertial.

Now let’s consider the following definition of inertial frame: An inertial frame is one in which Newton’s second law holds. Under these conditions the argument is:

Premises:

1) The change in the state of motion of an object is proportional to the applied force acting on it and points in the same direction (Newton’s second law).
2) An inertial frame is one in which the second law holds (definition proposed).
3) Newton’s second law holds in the frame selected (empirical data).

Conclusion:

The selected frame is inertial.

In the case of the first argument, where Newton’s first law is used as one of the premises, it should be pointed out that the argument is circular in the sense that to determine the degree of inertial motion of the reference frame the validity of the law has to be supposed. But the problem is that this may not be confirmed unless one is sure that the selected frame is inertial. The second argument points to the same difficulty since in the determination of the degree of inertial motion of a certain frame the validity of the second law must be supposed and, at the same time the validity of the law depends on the proper selection of the inertial reference frame. Even though it seems that both proposals are equally valid, there is a reason that favors the selection of the second option.

Specifically the principle of logical independence of the axiomatic body of a theory is violated by selecting the first option since there is no logical sense in recurring to the first law because it is not needed. The first law is a special case of the second law. Also, the second law is more general, and thus, logically more powerful.

Also, Newton seems to justify the use of the second law in the determination of inertial frames as the second quote of the Principia presented before states. Methodologically he also seems to favor this option since there is, in the application of the theory to specific cases, no explicit use of the first law. In fact Newton does not allude to it after its introduction at the beginning of the *Principia*.

Other proposals may be examined for defining the concept of 'inertial reference frame' by filling the missing premises in the following deductive argument:

Premises:

1) A body persists in a state of constant velocity unless there is a force that alters this motion (Newton's first law).
2) (your definition of inertial frame)
3)
4)

…

n) Newton's first law holds in the frame selected (empirical data).

Conclusion:

The selected frame is inertial.

Even though there is a possibility of finding other definitions that may take us to the conclusion, it is my hypotheses that they will not free the reasoning from the violation of the principle of logical independence of the axiomatic body or from circular reasoning.

7. Final Remarks

The fact that there is circularity in the sense that there is no way to confirm the law, in both cases, without assuming that the frame is a Newtonian mechanics inertial frame but, at the same time there is no way of confirming that the frame is an inertial one without supposing the validity of the law, is an example of what happens to every empirical theory. Theories are tested in their own context. In this sense all we can ask from a theory is consistency. This methodological aspect in Newton's theory is validated by Newton himself when in the *Scholium* presented after the *Definitiones*, before presenting the laws of motion and how they work in particular cases, he declares the following: "*Motus autem veros ex eorum causis, effectibus & apparentibus differentijs colligere, & contra, ex motibus seu veris seu apparentibus, eorum causas & effectus, docebitur fusius in sequentibus*"[7] (*Newtono*). These limitations are not particularly characteristic to Newton's theory but to every empirical or factual theory and reflect the hypothetical nature of all scientific theoretical knowledge.

Other approaches have been proposed in order to justify the axiomatic nature of the law of inertia. Perl declares:

"From the evidence cited for the Laws, as contrasted with their origin in primitive experience or common sense observation, their mathematical character becomes further confirmed. For the Laws are said to agree with experimental laws, experimental theories, with each other, and with related principles" (Perl). However, the fundamental nature of an axiom, as has been discussed here, lies in its logical independence from other statements presented in the deductive structure that is the theory and not in their ability to confirm. If the information provided by a law is obtained deductively from other statements already introduced within the theory, then the deduced law is not an axiom but a theorem in the theory. In this paper Perl recognizes that "the first Law is a limiting case of the second" (Perl).

It must be clarified that the discussion presented in this paper points to the problem of adequate theoretical representation of scientific theories and this aspect lies within the field of philosophy. The critic I present does not intend to change scientific methodology. In the field of science there is a series of permitted procedures that are completely justified as part of the work of scientists. This is all acceptable.

[7] "How are we to collect the true motions from their causes, effects, and apparent differences; and *vice versa*, how from the motions, either true or apparent, we may come to the knowledge of their causes and effects, shall be explained more at large in the following tract" (Newton).

The fact that Newton's first law is not an axiom of Newtonian mechanics does not prohibit scientists to recur to it in any way they desire. There should be no problem in accepting that this law may be preferred, for example, to get a more clear perspective of what an inertial frame is or to make it easier to understand the concept of inertia. However, since the problem of the formal and correct representation of scientific theories lies within the field of philosophy in which accepted methodologies and conclusions differ from those in science, it is of vital importance to analyze theory representation on these rather more strict parameters.

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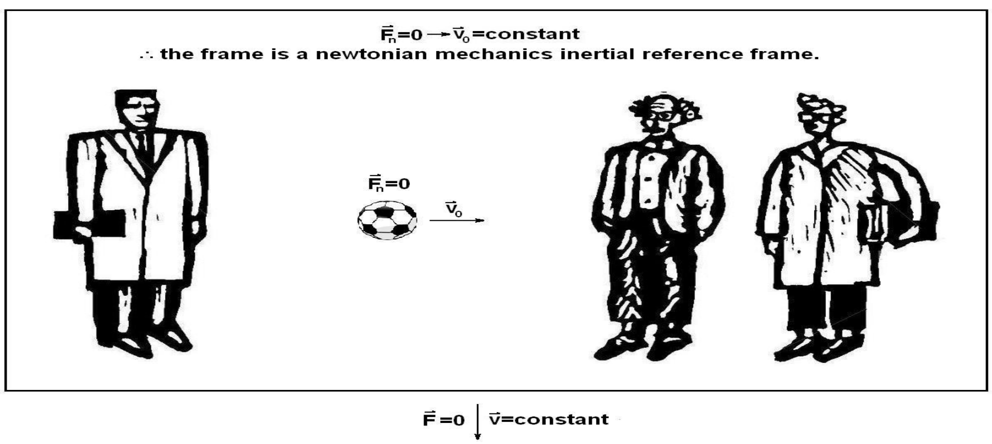

Figure 1: Experimental Confirmation of Newton's First Law when the Whole System is in a State of Motion with Constant Velocity

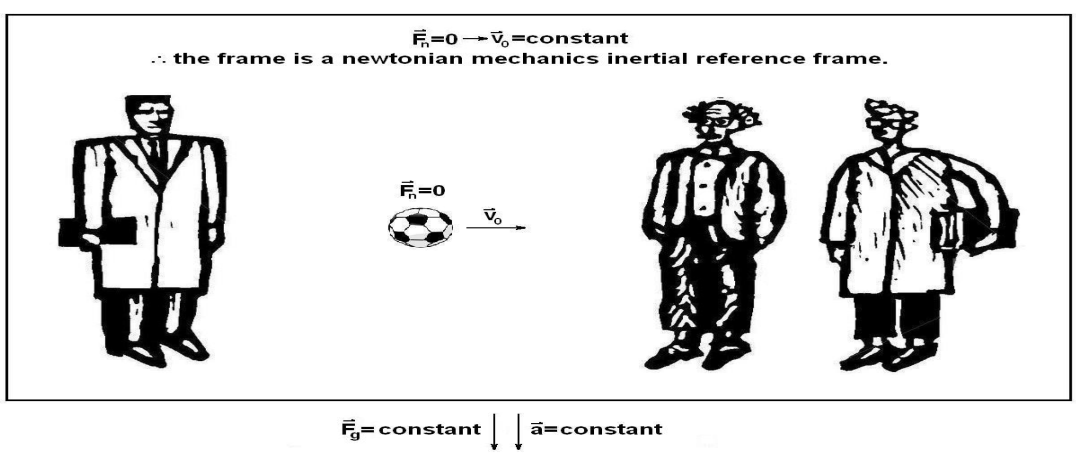

Figure 2- Experimental Confirmation of Newton's First Law When the Whole System is in a State of Free Fall Due to a Gravitational Force Acting on it

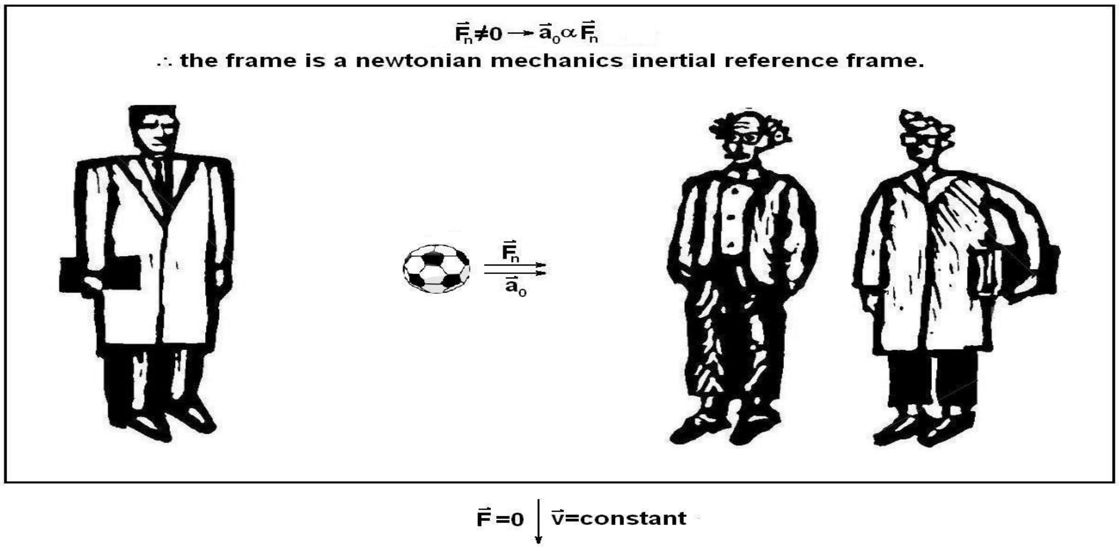

Figure 3- Experimental Confirmation of Newton's Second Law when the Whole System is in a State of Motion with Constant Velocity

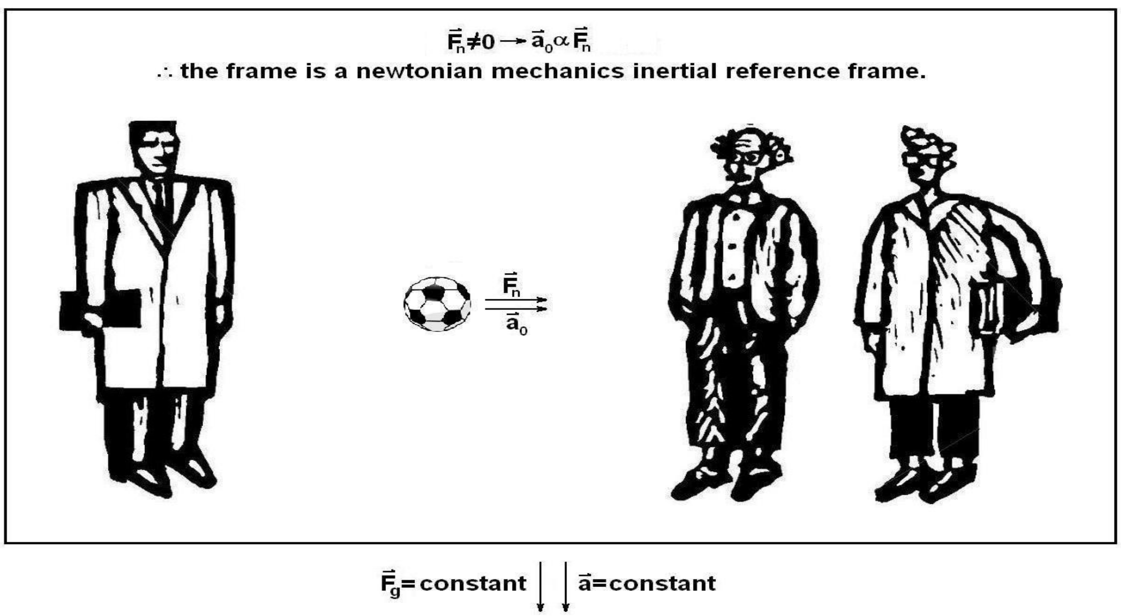

Figure 4- Experimental Confirmation of Newton's Second Law when The Whole System is in a State Of Free Fall Due to a Gravitational Force Acting on it