

# Celebrity Effects: How Famous Traders Impact the Financial Market<sup>1</sup>

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## Abstract

*Imitation is one of those personal behaviors which have profound social and economical implications. It has been suggested that this phenomenon is the leading cause of wide spread modes and fashions.<sup>3</sup> Even financial markets with rational, and to some extent, experienced and serious participants are not immune from imitative behaviors. The term "animal spirit" was used by Keynes mostly in reference to these kind of behaviors. In this paper, we study a model of financial markets in which there is a "star" trader whose actions are watched and are given a special group of traders which I call them "star-watchers". We investigate the price formation in this model in which a single asset is being traded, and two types of traders star-watchers, and "news-watcher", trade and in tract to form the market comes to affecting other people's behavior. We show how this celebrity or star effect can inflate prices and be a cause of bubble formation in the financial markets.*

**Key Words:** Star trader; Star-watchers; News-watchers; Celebrity-effect; Market maker; Asset prices, Bubble.

**JEL classification:** G14, D83. 1.6

## 1. Introduction

The actions and opinion of celebrities, and public opinion leaders have a special effect on their fans and on the society they live in. In politics, business and other aspects of social life, celebrity endorsement has a positive effect to sway the opinion of others. For example, celebrity endorsement is a big business in the marketing industry. Advertisement campaigns have been paying great sums of money to celebrities to endorse, or even just to use, their products. The best sign that these kinds of endorsements are beneficial is the amount of money that companies spend on celebrity endorsement, a practice that shows no sign of slowing down. For instance, in Forbes magazine's (2004) lists of the top 100 celebrities Golfer Tiger Woods, ranks number 3 and has a \$105 million dollar contract with Nike. "Several studies have examined consumers response to celebrity endorsements in advertising, findings show that celebrities make advertising believable." (Jagdish & Wagner 1995) and "advertising uses celebrities as pioneers in order to dictate trends". Also, studies have shown positive relationships between the stock price and the usage of celebrity endorsement in the advertising strategies of a company<sup>4</sup>.

One of the questions which I try to answer in this paper is the effect of imitation, in financial markets. In other words, is the price mechanism in stock and other financial markets able to convey information efficiently in such a way that diminishes the celebrity status of famous traders? Numerous cases can be mentioned as evidence that prices lack such ability. For example on Wednesday September 16, 1992, a day that is remembered as *Black Wednesday*, George Soros almost single-handedly forced the British government of the day to abandon the European Exchange Rate Mechanism. Besides yielding him almost one billion US dollars, this incident hugely enhanced his reputation too, so that in April 1993, when he bought around 3 million ounces of gold at \$ 345 per ounce and invested \$ 400 million in Newmont Mining-a gold mining company, as soon as the traders learned of Soros' purchase, gold rose \$ 5 after a long period of decline, a trend that continued to 1996 and lifted the price of gold to \$ 405. His investment in British real estate, which subsequently skyrocketed the price of real estate, and the Malaysian prime minister's accusation that Gorge Soros has ruined the East Asian economies-in reference to the 1997 crisis in East Asia - are other examples of how much influence a single trader can have on other traders' behavior and subsequently the market as a whole.

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<sup>3</sup>See for example, "Bikhchandani S., Hirshleifer D., Welch I. 1992," and references therein.

<sup>4</sup>For example see "Srivastava et al" Journal of Marketing 1998 and the references therein.

More recently, after the market crash of 2000, the United States Congress held hearings entitled "Analyzing the Analyst" aimed at addressing stock analysts and their recommendations, suggesting that words and recommendations can have a huge impact on the behavior of other participants. Also in 2002 the NYSE and NASDAQ issued new regulations, which were primarily aimed at the top ten investment banks, usually called big tens, to curb the conflicting interests on the analysis and recommendations issued by the big banks and famous analysts. Some even suggested that there has been a conspiracy to push the market up by frequently issuing very positive recommendations. Titles like "Wall Street treachery: leading the lambs to the slaughter" or "The betrayed investors: American bought to the idea that stocks would only make them richer" (both from Business Week) suggest a more intentional misleading.

This paper is related to the works on herd behavior in the sense that we try to see how imitation and herding change the process of price formation in the financial markets. Unlike the works in herd like Banerjee (1992), and Bikhchandani et al (1992), we do not analyze the motivations behind the herding. In this paper we consider the existence of the star trader and star-watchers as a fact and then see how this phenomenon effect the prices.

The argument made by Banerjee (1992) and Bikhchandani et al (1992), from now on BHW, shows that herding is not necessarily an irrational phenomenon. These papers argue that, if people act in sequence and observe the actions of their predecessors without accessing the actual information received by them, the information contained in the history of actions eventually will overwhelm the private information of every agent forcing them to abandon their own private information and follow the actions of their predecessors. BHW also argue that their model can be a base for understanding the uniformity of social behaviors and the creation of norms and fashions. Avery and Zemsky (1998) have shown that while it might be the case when the cost of choosing different actions is fixed, the argument breaks down in the presence of an adjustable price. Therefore the price mechanism in financial markets will adjust in such a way that every participant will be better off following his own private signal. They show that in order for herding to happen we need what they call multidimensional uncertainty.

While Avery & Zemsky (1998) suggest that informational herding is a very rare phenomenon, other sources of herd behavior might still exist. There is a large literature in reputation-based herding. Scharfstein and Stein (1990), Trueman (1994), Zweibel (1995), Graham (1999) and others provide another theory of herding in financial market based on the reputational concerns of fund managers or analysts.<sup>5</sup>

In this paper, we introduce two types of traders, "news-watchers", and "star-watchers". our traders are boundedly rational. Star-watchers are so because they consider the star's action as more informative than they really are. Although star traders might have a higher ability to process information, but very well it might be pure luck which has made them stars, news-watchers are boundedly rational because they don't discount the possible effect of the star-watchers. In the following sections we first introduce the model, then we study the trading behavior and price setting of the news-watchers and star-watchers. The market is the place which all types of traders interact. We consider markets with different make ups and will study how prices form, and deviate from the fundamental evaluation. In the end we provide some simulations based on the model presented in this paper.

## 2. The Model

The model we introduce here, has two types of traders. The first type which we call them *news-watchers*, and the second type which we call them *star-watchers*. As in AZ and other similar papers, I assume that trades take place in a sequential manner. The first trader observe a private signal and then buys, sells, or hold to her positions and then the second trader does the same, then the third, and so on.

We consider a model in which the market is for just one single asset with true value  $V$  in such a way that  $V \in \{0,1\}$ . Prices are set by a competitive market maker who interacts with an infinite sequence of individual traders who are chosen from a continuum population. This assumption guarantees that no trader appears in the sequence more than one time. Thus, we need not to worry about strategic considerations. Each trader is risk neutral and has the option to buy, sell, or hold onto one unit of stock. Trades occur at dates  $t = 0,1,2, \dots$

There are two classes of traders in our model. Informed traders who receive private information and try to maximize their profit using their private, and possibly public information.

<sup>5</sup>For a survey of herding in financial market see "Herd Behavior in Financial Markets" by Bikhchandani and Sharma.

This class divides into two subclasses. "News-watchers" who follow strict Bayesian reasoning without putting any special weight on any particular traders, and "Star-watchers" who also use Bayesian reasoning, but put more weight on the action of a particular trader who we shall call *the star* trader. The second class of traders are "noise traders" acting for liquidity considerations.<sup>6</sup>

We let  $\mu < 1$  denotes the probability of an informed trader arriving at any given time  $t$ . Therefore,  $1 - \mu$  is the probability of a noise trader arriving. Furthermore, and for further convenience, we assume that noise traders buy, sell, or do nothing, with equal probability:  $\lambda = (1 - \mu)/3$ . The details of this model is numerated below.

1. Traders trade sequentially and are selected from a continuum pool of trades. Each trader receive a private signal and then acts. Her action space is buy, sell, and hold.
2. We put a probability measure on the space of all traders and normalize it to one. There are three types of traders *news-watchers*, *star-watchers*, and *noise* traders. The portion of informed traders (those who are not noise-traders) is  $\mu$ , the portion who are star-watcher is  $\gamma$ , and the rest are news-watchers. The population of agents is a continuum and every agent has a label in  $[0,1]$ . To choose an agent at time  $t$ , a random number,  $r$ , will be chosen from a uniform distribution on  $[0,1]$ . If  $r < 1 - \mu$ , then a noise-trader has been chosen, if  $r \in [\mu, \mu + \lambda]$  a star-watcher has been chosen, and otherwise a news-watcher has been chosen. The law of large numbers guarantees that in each time  $t$ , noise traders, news-watchers, and star-traders appear with probabilities equal to what we assigned them above.
3. At time  $t$  a trader is selected randomly and after receiving a private signal she acts.
4. *News-watchers* follow the news and update their opinion based on the news (signals) they receive. In the other words news are the only thing that they follow and that is the only think that can persuade them to buy, sell, or hold. They update their opinion about the price using Bayes' rule.
5. *Star-watchers* follow the news, but they watch too see what other famous traders do. To model this we assume there is a *star trader* whose actions influence the opinion of star-watchers.<sup>7</sup>
6. There is one asset to be traded. The value of this asset is zero or one with equal probability. Before trading, each trader get a signal regarding the value of the asset at that time. The signal is in the form of  $p(V = 1) = \pi$ .
7. To clearly define the difference between news-watchers and star-watchers, we consider two different probabilities according to which they associate probability to the same event. Suppose that the "star trader" appears at time  $t$ ; if the trader at time  $t + 1$  is a star-watch, she assigns
 
$$P_s(V = 1|S = buy) = \pi^*, \quad (2.1)$$
 as the probability while if a news-watch trades at the time  $t + 1$  he assigns
 
$$P_n(V = 1|S = buy) = \pi \quad (2.2)$$
 such that  $\pi^* > \pi$ .
8. The Market: The market consists of these two types of traders plus some noise traders. The price determined by a market maker which consistently adjust the price to clear the market.

• **News-watchers' trading decisions:** As we explained above, the news-watchers' opinions are not effected by the trading actions of the star trader. They each receive a signal (news) and then decide what to do. Since prices adjust and convey the past information, there is no chance that news-watchers ignore their signal and fallow the star or the herd. Therefore a news-watcher will do the following: If  $P_n(V = 1|news) > p$  will buy, if  $P_n(V = 1|news) < P$  will sell, and if  $P_n(V = 1|news) = p$  will hold.

• **Star-watchers' trading decisions:** The star-watchers trading decisions are more complex. The action of the star has the power to change the opinion of these traders against their signal, meaning there are times that they ignore their signal and follow the star in a direction opposite to what the news indicates. In the following subsection we investigate we investigate the buy, hold, and sell decisions of the star-watchers. Putting the star-watchers and news-watchers together will generate the whole market. We see that the important that point here is the proportion of each group of traders.

<sup>6</sup>In the absence of noise traders, the no-trade theorem of Milgrom-Stocky(1982) applies and the market breaks down.

<sup>7</sup>We notice that there is no assumption indicating that the "star" has indeed access to better information nor that his signal is more accurate than others. Although it might be the case in the real world that famous people have such information, fans, anyway, frequently put too much weight on the star's actions. This model can be considered an attempt to capture such over reactions by the fans.

## 2.1 The Prices Process in The Market

Given the model in the last section, here we investigate whether if mispricing and bubbles can occur. To this ends we define

$$f(x) = \frac{px}{px + (1-p)(1-x)}, \quad (2.3)$$

and

$$g(x) = \frac{(1-p)x}{(1-p)x + p(1-x)}. \quad (2.4)$$

Let

$$n = \min\{m | g(\pi^*) \leq f^m(p)\}. \quad (2.5)$$

Then we have the following.

**Lemma 1** Let  $\bar{\beta} = f^n(p)$  and  $\beta = g^n(p)$ , where  $n$  is given as above. Then, the size of any bubble is bounded from above by

$$\delta = |\bar{\beta} - \beta|$$

Another question that arises is that of how long it takes for the price of the asset reach to its highest level. The next proposition attempts to answer this question.

**Proposition 1** Let  $\pi^* = P_f(V = 1|H_0)$ ,  $p = P_{nf}(V = 1|H_0)$ , and  $n$  taken from proposition 3. Let  $T$  denote the time it takes for the price of the asset to reach  $\delta$ . We have the following.

- (a) If  $\gamma \leq 1/6 + 1/3\mu$ , then  $Prob(T < \infty) = 1$ , but  $\mathbb{E}[T] = \infty$ .  
 (b) If  $\gamma < 1/6 + 1/3\mu$ , then

$$Prob(T < \infty) = \left(\frac{\gamma - 1/3\mu + 1/3}{2/3 - \gamma + 1/3\mu}\right)^n < 1. \quad (2.6)$$

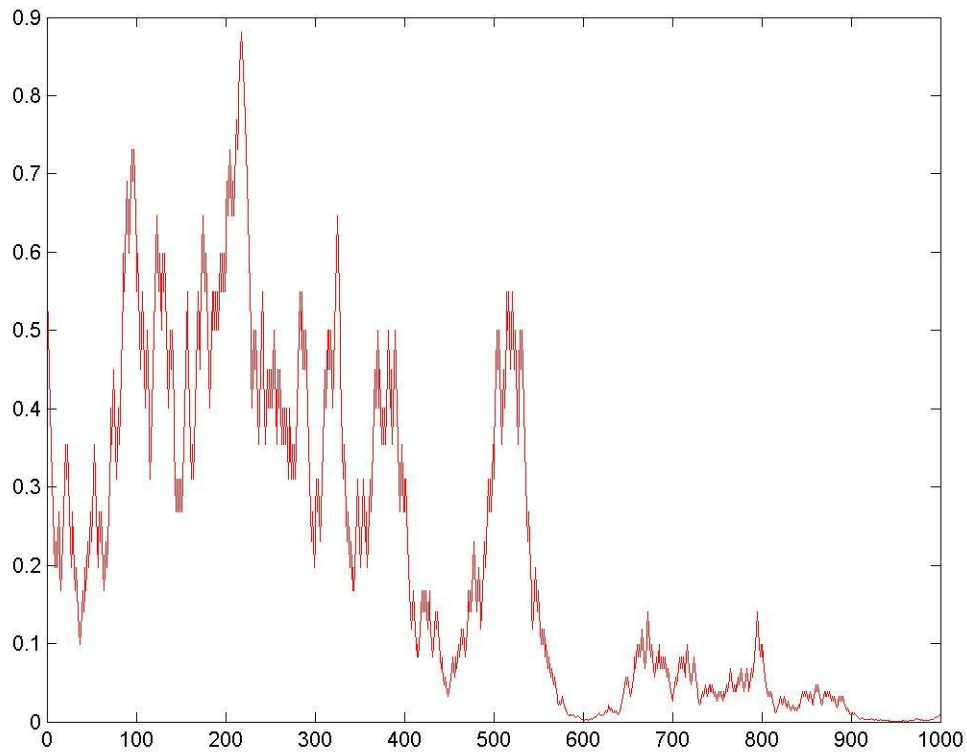
- (c) If  $\gamma > 1/6 + 1/3\mu$ . then

$$\mathbb{E}[T] = \frac{3n}{6\gamma - 2\mu - 1}. \quad (2.7)$$

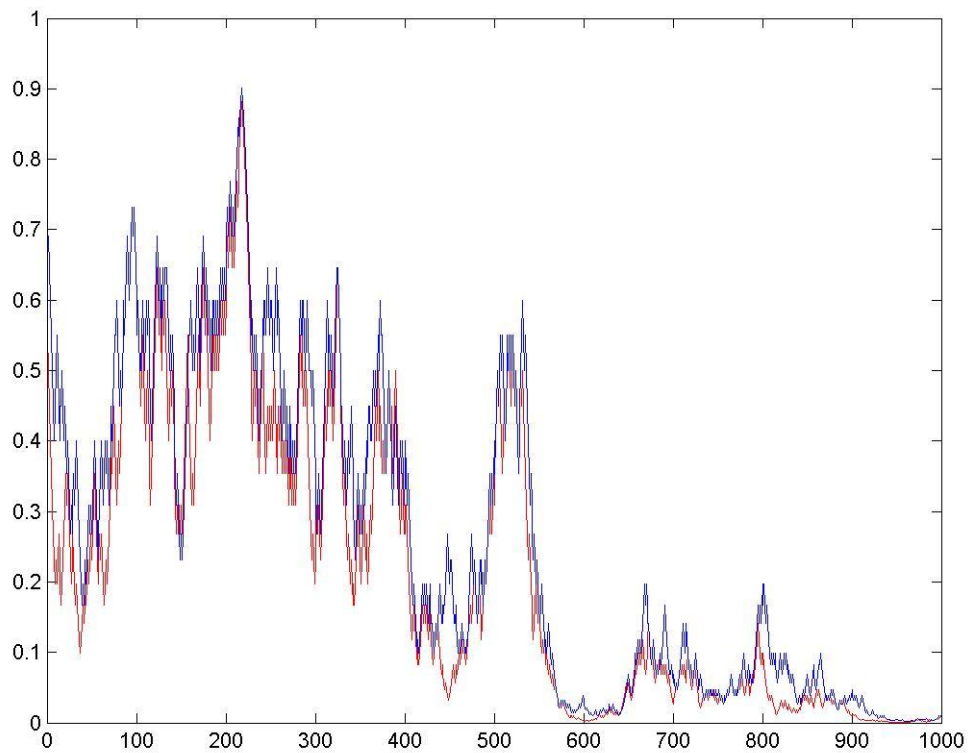
The difference between  $f(p)$  and  $g(p)$  in proposition 5 is not very large. This implies that  $(\bar{\beta} - \beta)$  won't grow too large. Therefore, when  $\pi^*$  (the primary faith of fans on the star) is not too high, the size of any bubble won't grow very large. Furthermore, proposition 6 suggests that it would be difficult for the price to "grow out of control". Additionally, when there are enough traders who don't follow the star, it is almost impossible to obtain a bubble in which the asset is substantially mispriced. The only time that we can expect these kind of bubbles to appear is when fan traders are dominating the market, so that a substantial portion of market participants are positively biased toward the star trader.

I have simulated the model discussed in this section. Figure 1 shows a sample path of the real price as implied by the model. We can observe from figure 2 that there won't be any substantial mispricing when we have enough normal traders to "time" the market. However, as figure 3 shows, in times when the fan traders dominate the market, 60% in this case, there is a good chances that we see bubbles particularly in bad times when the actual price should be falling. Both in this paper and in the simulations I have assumed that there is no changes of opinion, and that the fan traders have a fixed biased toward the star. A good exercise would be to alter the model so that in every period a participant is assigned a type which indicates whether the participant is a fan, and if she is, how biased she is towards the star. In this case, we can study situations in which the fan traders eventually will alert their trust on the star if the market is not going well in the direction that the star recommends. In order to do so, we need a model for this alternation. In other words, we need a theory that tells us how people alter their beliefs in critical times.<sup>8</sup>

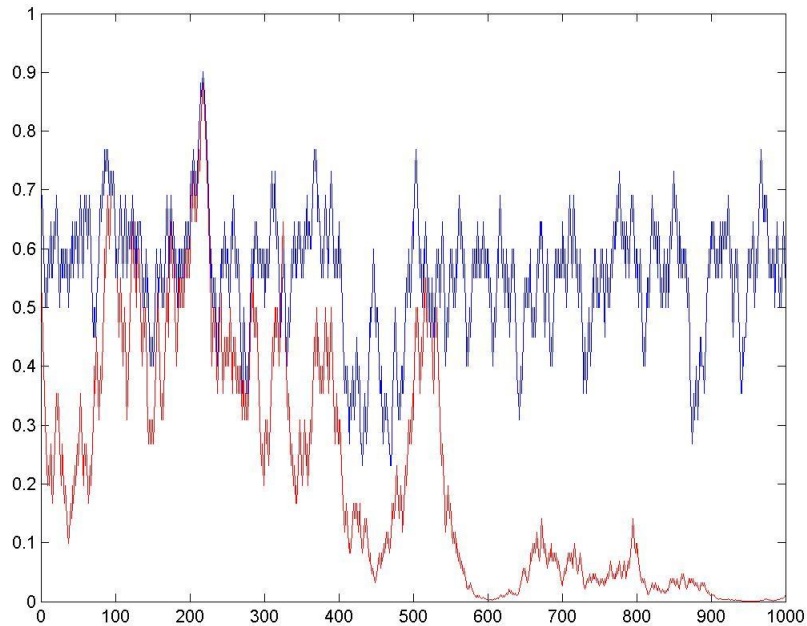
<sup>8</sup>If we just assume a random alternation of beliefs, I suspect that we won't get substantially different results from the simulations presented in this section.



**Figure 1: A sample path of the real price as implied by the model**



**Figure 2: When fans are 30%, noise traders are 10%, and normal traders are 60% of the total market**



**Figure 3: When fans are 60%, noise traders are 10%, and normal traders are 30% of the market**

## 2.2 A Possible Extension

In the previous section, we studied a case in which the star appears once at the beginning and, because some of the other agents consider her action to be more informative, they are willing to pay more for the asset than what their own signal recommends. This causes the price to be higher and a bubble is created.

It is worth noting that so far we have not assumed that the star investor has indeed access to special information which gives her the actual ability to make better decisions. While it might be the case in the real world that big investment firms have both better information and better ability to process this information, this model can be taken to suggest that inexperienced traders may exaggerate those abilities and subsequently put more weight on the stars' actions, more weight than the star action actually deserve.

An interesting question arises. What would happen if the star investor in our model can trade more than once? Is it possible that she starts to follow the herd which she herself has helped to create, and if so, what will be the size of a possible bubble created in this manner?

One possible way to answer these questions is that we assume that, unlike other traders, the star trader can indeed enter the market frequently. Furthermore, we can assume that the trust of her fans won't decrease nor increase after each entry.<sup>9</sup>

Now suppose that, for some exogenous reason, the star investor starts following the herd. For instance, we can think of a situation in which the star trader indeed does not get any informative signal, but is just summing up the information which is being revealed by the price and announces her choice to the public. I conjecture that large bubbles can exist in this scenario. This would be an example of a situation in which already publicly available information can have a large impact. Simply because the information is being announced by the star, her fans overreact to that information. The diagram below explains this idea.

(0,2)The Star acts

<sup>9</sup>In real world cases the trust or belief in the star will change from time to time. Imagine, for example, an investor who follows a recommendation and makes good money. It is quite possible that next time around he will follow the star's recommendations with more confidence.

### 2.3 An Example

As an example, suppose a competitive group of market makers, or equivalently a market maker who makes zero profit, determine the prices, by setting bids and asks prices as

$$B^t = E[V|h_t = S, H_t], \tag{2.8}$$

and

$$A^t = E[V|h_t = B, H_t]. \tag{2.9}$$

Here,  $S$  stands for selling orders and  $B$  stands for buying ones. We only analyze the buying activities (selling is similar). Therefore, we focus our attention on the prices at which the agents are willing to buy the asset. (See Lawrence Glosten and Milgrom (1985)). Suppose that there is a star investor such that his decisions are observed by all other investors. There are also news-watcher, and star-watchers who do not observe each others decisions. These assumption have been made to simplify the calculations and the computer simulations we perform. We also assume that star-watchers consider the actions of the star investor to be more informative than their own as we discussed it above, and that the star investor enters at the beginning. Every buyer receives a private signal  $x \in \{0,1\}$ , s.t.  $P(x = V) > 1/2$ . Suppose that the prior probability of  $V = \{0,1\}$  is  $P(V = 1) = P(V = 0)$ . Given this information, we can find the probability of the value being equal to one if the star investor buys.

$$\begin{aligned} P(V = 1|h_{star} = B) & \tag{2.10} \\ &= \frac{P(h_{star} = B|V=1)P(V=1)}{P(h_{star} = B|V=1)P(V=1)+P(h_{star} = B|V=0)P(V=0)} \\ &= \frac{P(h_{star} = B|V=1)}{P(h_{star} = B|V=0)+P(h_{star} = B|V=1)} \\ &= P(h_{star} = B|V = 1) = \pi_1. \end{aligned}$$

Here,  $\pi_1$  is the probability that  $V = 1$  if the star investor buys. We have assumed that  $\pi_1 > p$ , which implies that other agents consider the star's information more accurate.

Now, suppose that at time  $t = 0$  the star investor buys. The market maker will set the price for  $t = 1$  to be

$$V_1^m = E_m[V|h_0 = B] = P(V = 1|h_0 = B) = p. \tag{2.11}$$

At the same time, a star-watcher who gets a negative signal at time  $t = 1$  will evaluate the price as:

$$\begin{aligned} V_1^A &= E[V|h_{star} = B, x = 0] = P(V = 1|h_{star} = B, x = 0) \tag{2.12} \\ &= \frac{(1-p)\pi_1}{(1-p)\pi_1+p(1-\pi_1)} = \pi_2 \end{aligned}$$

Now, if  $\pi_2 > p$ , the star-watcher will buy despite receiving a negative signal. The important observation is that this situation can indeed happen. Figure 1 describes a simulation with  $p = .52$ ,  $\pi_1 = .75$ , and  $\pi_1 = P(V = 1|h_{star} = B)$ . The probability that the star-watcher initially assigns to the event that  $V = 1$  when he sees the action of the star is assumed to be  $\pi_1 > p$ . As we see it takes a while (6 periods in this case) for the agents with negative signals to stop buying.

To illustrate this point better we repeat the process one more period. Now suppose that at time  $t = 2$  the agent whose turn is to act again receives a negative signal ( $x = 0$ ). The market maker will set the price:

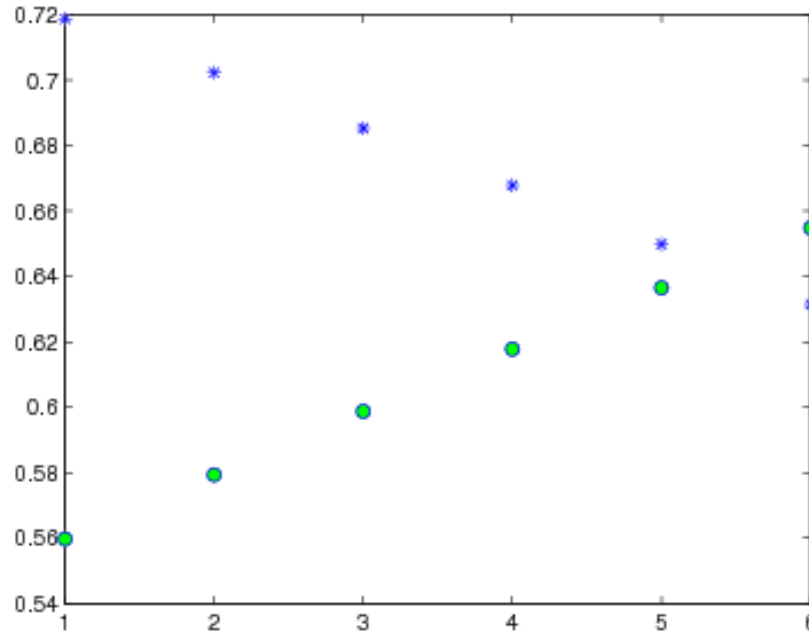
$$V_2^m = E[V|h_0 = B, h_1 = B] = \frac{pV_1^m}{pV_1^m+(1-p)(1-V_1^m)}. \tag{2.13}$$

While the agent's value is:

$$V_2^A = \frac{(1-p)\pi_2}{(1-p)\pi_2+p(1-\pi_2)}. \tag{2.14}$$

Again, if  $V_2^A > V_2^m$ , the agent will buy even though he has a negative signal. Thus, herding can happen in this situation. However, it will be short lived.

The important point to notice is that the market maker and agents use two different measures for evaluating the relevant probabilities.<sup>10</sup>



**Figure 4:** The stars show the prices as they are set by the star-watcher. The circles represent the prices set by the market maker. The horizontal axes shows the number of periods.

### 3. Conclusion

In this paper we studied a market in which a special trader has star status, meaning that she has her fan and followers, which we called the star-watchers. We showed, that while the market mechanism can prevent herd behavior from happening in a very simple setting, it will fail to do so when the herd behavior is the result of a more complex belief system in which a star and her fan's are presented.

One of the implicit implications of our study is that it suggests that a rise or fall in prices of stocks of big investment banks may have a broader impact on the entire market. This is because, besides the real effects that change in the price of a particular stock might have on the market, a rise or fall in the price of stocks of the investment banks will have the additional effect that the investors who have been following these firms (being fans in our terminology) will revise their belief on the accuracy of the information of these firms. For example, in the case of a price fall, the fan investors might put much less weight on the recommendations given by their star or even revisit their previous investment decisions which were done in accordance to the actions previously taken by the star, resulting in a further decline. To give a measure of herd behavior or to determine when herding is happening, is difficult.<sup>11</sup> However, it is possible to measure and test the correlation of stock prices with the movements in the stock price of big financial firms, specially in times of bubbles.

This study also might be able to shed some light on the question of why announcements of already published information sometimes have a substantial effect on the stock prices, since if the re-announcement is done by a star trader, according to our model, will have an extra effect on the star-watchers and through them it can effect the market. Another implication of our study suggest that when there are a lot of inexperienced traders in the market, and the sources who are trusted by the public fail to provide carefully crafted and implied analysis, and instead they themselves are being driven by the public's actions, the probability of crisis is very high.

<sup>10</sup>We conjecture that the price that market maker sets is still a martingale with respect to the market maker's measure. This is intuitively obvious since if it was not a martingale, then his assessment of  $V_t$  would be systematically mistaken in a manner which should be predictable to him.

<sup>11</sup>See Bikhchandani and Sharma (2000) for references.



**Appendix**

Proof of Lemma 1:

*Proof.* Because non-fan traders follow their own signal, their participation helps to control any miss-pricing. Therefore, in order to find an upper bound for any possible bubble, we can assume that all traders are fans.

Suppose everybody receives a negative signal but after weighting in her/his initial belief decides to buy. How long can this process continue? As soon as  $g(\pi^*) \leq f^m(p)$ , the  $m^{th}$  trader stops buying. Therefore, the length of the buying process is

$$n = \min\{m | g(\pi^*) \leq f^m(p)\}. \tag{3.1}$$

The next step is to investigate how much a bubble can grow during these n periods. If the market maker could see the actual signals he would have set the price according to  $\beta = g^n(p)$ . Since, he cannot see the actual signals and he only observes the "buy" and "sell" actions, he increases the price according to  $\bar{\beta}$ . Therefore, the size of the bubble is

$$\bar{\beta} - \beta \tag{3.2}$$

Proof of Proposition 1:

*Proof.* In the proof of proposition 3 we assumed that all traders are fans, which implies that no correction takes place and the size of any possible bubble rapidly grows until it reaches the established upper bound. Now, if we take into consideration the presence of noise traders and non-fans, we are going to have an asymmetric random walk on  $\mathbb{R}$  which moves up and down with different probabilities depending on the combination of fans, non-fans, and the noise traders. The following lemma is the core part of the proof.

**Lemma 2** Let  $X_1, X_2, \dots$  be i.i.d with

$$P(X_i = 1) = p \text{ and } P(X_i = -1) = 1 - p \text{ } p > 1/2$$

and let

$$S_n = X_1 + X_2 + \dots + X_n \quad \alpha = \inf\{n: S_n > 0\} \quad \beta = \inf\{n: S_n < 0\}.$$

Then,

- (i)  $P(\alpha < \infty) = 1$  and  $P(\beta < \infty) < 1$ .
- (ii) If  $Y = \inf S_n$ , then  $P(Y \leq -k) = P(\beta < \infty)^k$ .
- (iii)  $\mathbb{E}\alpha = \frac{1}{2p-1}$ .

*Proof.* Sketch of a proof:

(i): We need the following result for the proof of this part this can be found as theorem in "Probability: Theory and Examples by Richard Durrett."

**Theorem 1** For a random walk on  $\mathbb{R}$  there are only four possibilities, one of which has probability one.

- (1)  $S_n = 0$ , for all n.
- (2)  $S_n \rightarrow \infty$ .
- (3)  $S_n \rightarrow -\infty$ .
- (4)  $-\infty = \liminf S_n < \limsup S_n = \infty$ .

We also need the following statement in the proof.

Let  $\alpha$  and  $\beta$  be the same as above. Then the four possibilities of the theorem correspond to the following four combinations  $P(\alpha < \infty) < 1$  or  $= 1$  and  $P(\beta < \infty) < 1$  or  $= 1$ .

Part (i) of the lemma can easily be derived from the fact that

$$P(\beta < \infty) < P(\alpha < \infty). \tag{3.3}$$

(ii): This part is obvious when we consider that the  $S_i, s$  are independent, and  $Y \leq S_i, \forall i$ .

(iii) A result in stopping time theory -sometimes referred to as Wald's equation- states that:

If  $X_1, X_2, \dots$  are i.i.d with  $\mathbb{E}|X_i| < \infty$ , and if  $\tau$  is a stopping time with  $\mathbb{E}\tau < \infty$ , then:

$$\mathbb{E}S_\tau = \mathbb{E}X_1 \mathbb{E}\tau. \tag{3.4}$$

Apply Wald's equation to the stopping time  $\alpha \wedge n$  and let  $n \rightarrow \infty$  to obtain:

$$\mathbb{E}\alpha = \frac{1}{\mathbb{E}X_1} = \frac{1}{2p-1}. \quad (3.5)$$

The only thing that remains is to calculate the probability of a "buy" which moves the price up. This probability is  $1/3(1 - \mu) + \gamma$ . Now to prove part (a), notice that when  $\gamma = 1/6 + 1/3\mu$  the  $1/3(1 - \mu) + \gamma = 1/2$ , and we have a symmetric random walk in which  $\text{Prob}(T < \infty) = 1$ , and  $\mathbb{E}[T] = \infty$ .

For part (b), if  $\gamma < 1/6 + 1/3\mu$ , then  $1/3(1 - \mu) + \gamma < 1/2$  and, therefore, we have an asymmetric random walk, thus, by part (i) of lemma 3,  $P(T < \infty) < 1$ . In additions, by part (ii) of the lemma 3,

$$P(T < \infty) = P(\beta < \infty)^n. \quad (3.6)$$

For part (c), notice that if  $\gamma > 1/6 + 1/3\mu$ , we have an asymmetric random walk with the probability going up greater than the probability of going down. By part (i) of lemma 3,  $\text{Prob}(T < \infty) = 1$  and by part (iii) of lemma 3, we have

$$\mathbb{E}[T] = \frac{n}{2p-1}, \quad (3.7)$$

where p is the probability of going up.

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